A Model of Intermediation, Money, Interest, and Prices*

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Abstract

A model integrates a modern implementation of monetary policy (MP) into an incomplete-markets monetary economy with wage rigidity. MP sets corridor rates and conducts open-market operations. These tools grant independent control over credit spreads and inflation. We study the implementation of spreads and inflation via different MP instruments. Through its influence on spreads, MP affects the evolution of real credit, interests, output, and wealth distribution (both in the long and the short run). We decompose effects through different transmission channels. The combination of incomplete markets, wage rigidity, and a zero-lower bound on deposits, introduce a trade-off between micro insurance (redistribution) and macro-insurance (stabilization). As a result, MP should balance operate with a small balance sheet that induces positive spreads during booms, but expand its balance sheet during recessions.

Keywords: Monetary Economics, Monetary Policy, Credit Channel.

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1 Introduction

In modern economies, MP operates through the provision of reserves and a corridor of policy rates.[1] A popular view among academics is that these tools implement a desired nominal interest rate, which grants control over inflation, and this is ultimately what matters for MP (Woodford, 1998). A bank-centric view has it that these tools influence bank credit and spreads, and thus, impact real activity through their influence on the financial system (Bernanke and Blinder, 1988, 1992). Although this view is widely held by practitioners, and has strong empirical support (Kashyap and Stein, 2000; Drechsler et al., 2017), its theoretical foundations are still being laid out. This paper presents an incomplete-markets economy where credit is intermediated by banks that hold reserves to manage liquidity. MP is implemented through a corridor system and open market operations (OMO). On positive analysis, the paper articulates how these tools affect credit, monetary balances, borrowing and lending rates, inflation, and output, in the context of an incomplete-markets economy. On normative analysis, the paper articulates why the size of the central bank's (CB) matters.

In the present environment, operating a corridor system grants MP enough tools to implement an inflation target and manage credit spreads, as independent policy targets. In particular, interest on reserves grant control over inflation whereas the OMO the ability to move spreads. Whereas the control over inflation relates to well-traveled transmission mechanisms (the interest-rate and inflation-tax channels), the control over credit spreads is a notion of the credit channel.[2] This feature allows for the positive of the credit channel within an incomplete-markets economy. Studying the credit channel in an incomplete-markets economy is important because it fleshes out an important policy tradeoff. Through the ability to induce credit spreads, MP can limit the amount of micro-insurance in an economy. However, by inducing positive spreads, MP can limit the amount of credit in the economy, to provide better macro-insurance. This trade-off is in fact at the core of recurrent and historical debates (Bagehot, 1873; Stein, 2018) in which, during booms it is argued that MP *is sowing the seeds of crises,* but during busts, that MP is akin to *pushing on a string.* Our framework allows to articulate these views in terms of a trade-off between micro and macro insurance. We argue that to achieve the right balance, MP should operate with a small balance sheet (and positive spreads) during booms, but expand its balance sheet (and eliminating spreads) during busts. This

[1] A corridor system is a framework/procedure for implementing monetary policy whereby a central bank can use/combine various tools to steer the market interest rate toward a chosen target. Two important tools are the discount rate and the interest rate on reserves. The discount rate is the rate at which a central bank lends reserves, against collateral, to banks that are below their reserve requirement. The discount rate tends to be the upper bound or ceiling for the market interest rate. The interest rate on reserves is the rate at which banks are remunerated for holding reserve balances at the central bank. It tends to be the lower bound or floor for the market interest rate for interbank loans. These two rates form a "corridor" that will contain the market interest rate. Open market operations are then used as needed to change the supply of reserve balances so that the market interest rate is as close as possible to the target.

[2] A narrative description of different transmission channels of MP is found in Ben S. Bernanke (1995)'s "Inside the Black box." Kashyap and Stein (2000) presented evidence on the credit channel by exploiting differences in the cross section of liquidity ratios across banks. Bindseil (2014) describes the modern implementation of MP through banks across countries.

message is particularly pertinent now that countries are considering replacing corridor systems with floor systems and permanently switching to regimes with large balance sheets. For this paper, this change means surrendering an important policy tool.

We build this case through the study of a canonical continuous-time incomplete-markets environment. This is an endowment economy where households face idiosyncratic unemployment risk, as in Huggett (1993). This is where micro-insurance comes about. To speak to aggregate demands externalities, we introduce wage rigidities as to this incomplete markets economy, as and Kaplan et al. (2016). The novelty is the introduction of intermediation and money into this environment, which enables MP to control credit spreads. Credit is nominal and intermediated by a fringe of competitive banks. In addition to deposits and loans, banks hold reserves to manage liquidity. The power to influence spreads stems from an institutional feature. Whereas loans are permanently held by the issuer bank, deposits circulate. Thus, banks use reserves to settle deposit transfers. A potential shortage of reserves by some banks opens the door for interbank credit. The interbank market, however, operates with matching frictions (á la Ashcraft and Duffie, 2007; Afonso and Lagos, 2015). As a result, not all reserves deficits can be tapped with private credit and some deficits are forced to be borrow at a penalty rate set by MP. The overall quantity of reserves and the corridor rates set by MP translate into an intermediation cost. Ultimately, banks are a pass-through from a policy corridor spread to actual credit spreads.

A similar implementation of the credit channel already appears in Bianchi and Bigio (2017a), and in related work by Piazzesi and Schneider (2016); De Fiore et al. (2018); Chen et al. (2017); Drechsler et al. (2017). Here, bank decisions are simplified, and the pass-through from policy rates to spreads is immediate. The emphasis is not on the banking sector, but on the effects of spreads in an incomplete market economy with nominal rigidity. The latter delivers a broad set of implications for changes in credit spreads. Notably, the real effects of MP are driven by the precautionary motive. Because MP indirectly affects the distribution of wealth, it influences the mass of agents that choose the inefficient endowment, and this impacts productive efficiency. Because this mechanism is independent of inflation, the model connects transparently with other transmission mechanisms.[3]

The paper first delves into the details of implementation. It presents closed-form expressions for nominal deposits and loans interests. These nominal rates carry different premia over the rate on reserves. The difference between these premia is a real credit spread, which, in turn, is expressed as a function of a liquidity ratio and the policy corridor spread set by MP. The implementation is explicit about a reserve satiation regime (a floor system), and a zero lower bound on deposit rates (DZLB). Away from either regime, OMO and/or reductions in policy corridor spread, implement a reduction in the credit spread. Another tool, the interest on reserves, grants direct control over inflation, without affecting on the spread. In a satiation regime, all rates equal the interest on reserves,

[3]In the language of Achdou et al. (2019), a borrower-lender spread is dubbed "soft-constraint." The mechanics of the credit-channel can thus be interpreted as the ability of MP to affect soft constraints in an incomplete markets economy.

so MP can control inflation, but not spreads.[4] In a DZLB, OMO are irrelevant, but reductions in the interest on reserves can produce a joint movement in credit spreads and inflation, a phenomenon that has been recently identified in the date (Heider et al., 2019; Eggertsson et al., 2019).

After the implementation, the paper presents a positive analysis of the real effects of MP. The equilibrium in the economy is summarized by a single clearing condition; the deposit, money and loans markets collapse into a single market for real claims. In turn, clearing in the real claims market is influenced by a spread target. At steady-state, there is a single-real interest that clears the market in real claims. Thus, holding fixed a spread target, changes in nominal rates are super neutral. However, through its control over spreads, MP influences the long-run the real rates at which real claims clear. In particular, it lowers the real savings rate, but produces an even stronger increases in lending rates. This inhibits long-run insurance.[5]

In the short-run, due to nominal rigidities, the real rate is pinned down by the path of inflation and policy rates. Thus, the variable that clears the market for real claims is the rate of job separation. In the short, thus, changes in nominal rates operate through the interest-rate channel. In particular, reductions in the interest on reserves are expansionary, but only until the point where the DZLB is reached. Beyond that point, reduction in real rates are contractionary. Thus, changes in policy rates have effects on aggregate demand but only up to a point where the economy hits the DZLB. In turn, open-market operations in the short-run impact spreads, and through these, they affect aggregate demand, but their effect disappears after the economy hits a satiation limit. Thus, OMO have effects also through insurance and through this channel, on aggregate demand but only up also up to a point.

The model also has implications for the statistical relationship between monetary aggregates and inflation. Whereas the model is entirely consistent with the quantity theory of money, it can also produce a liquidity effect. For example, a temporary OMO can produce a reduction in inflation. The effect of the operation is a reduction in spreads and an increase in output, which increases the real deposit rate. If MP keeps the rate on reserves constant, the monetary expansion is deflationary.

Turning to the normative analysis, the optimal spread is governed by a trade-off between insurance and leaving "powder" for macroeconomic stabilization. The paper ends with a study of the problem of an egalitarian planner that expects the economy to suffer an aggregate shock such as a credit crunch shock or an impatience shock. Upon the shock, the can lower the interest on reserves, and

[4] Different from Woodford (1998), the control over nominal rates is achieved without OMO, but by setting the interest on reserves. Inflation changes are neutral, but we are explicit that with additional frictions, a control over nominal rates can produce effects through the interest rate, inflation cost, and debt deflation channels, all of which can be thought of as operating independently. In each case, the model would need an additional ingredient: nominal rigidities, cash transactions, and long-term debt, respectively.

[5] A permanent increase in the spread primordially impact the loans rate, the effect on efficiency is driven by the impact on borrowers. With higher loans rates, borrowers have stronger incentives to repay debts, on the one hand. On the other hand, debt rollover becomes more difficult with higher rates. As a result, mild increases in spreads concentrate the distribution of wealth toward the middle as borrowers repay their debts faster. Further increases in spreads, makes it harder to repay debt, to the point that borrowers give up. This effect fans out the distribution of debt, and more agents hit their debt limits.

can even reach negative territory. However, it will not want to do so beyond the point where it triggers a DZLB because beyond that point, the shock is recessionary. Because aggregate demand cannot be fully stabilized with an interest-rate policy, the planner wants a positive steady-state value for spreads which is closed after the shock. This positive spread, although limits the degree of social insurance at stead-state, limits the unemployment losses upon an aggregate shock. We find that the optimal steady-state spread is a positive spread that balances distributional considerations against efficiency considerations.[6] To implement the counter-cyclical spread, the CB must operate with a lean balance sheet at steady-state, but flood banks with reserves during a crisis. In other words, MP should operate a corridor system that satiates banks with reserves during crises, but runs through a standard corridor system in normal times.

Connection with the Literature Our paper's title emphasizes the connection with the two most common frameworks for MP analysis. One approach emphasizes the connection between *money and prices* and the other between *interest and prices*. In the first approach, money plays a transactions role (Lucas and Stokey, 1987; Lagos and Wright, 2005) and there is a tight connection between prices and the quantity of (outside) money. The real rate is fixed, so any real effects follow because inflation is a transactions tax. The second approach is the new-Keynesian approach where the important connection is between *interest and prices*. Under that framework, MP controls real rates directly because prices are rigid. There is no role for monetary balances. Neither framework emphasizes the effect of MP on credit, at least not directly. The model here establishes a meaningful connection between *intermediation, money, interest and prices*. Because the credit channel here can be studied independently of the control of inflation, it only complements the inflation-tax or interest-rate channels in those approaches.

Since 2008, there's been an increased interest in how MP interacts with credit markets. That gap is being filled, and incomplete market models are a natural starting point.[7] In fact, the first generation of heterogeneous agent models, Lucas (1980) and Bewley (1983), were about MP and were not interested in heterogeneity per se. However, neither model established how MP affects credit.[8] Credit, of

[6] In terms of redistribution: wider spreads hurt everyone, especially the very poor and very rich who care the most about rates.

[7] Models that feature credit must provide a motive for credit. One way is to endow agents with different technologies as in Bernanke and Gertler (1989) and the other is make them subject to idiosyncratic risk. To establish a connection between MP and credit markets, models must have features by which MP impacts credits. A first such model is Bernanke et al. (1999), which incorporated nominal rigidities into the two-sector economy of Bernanke and Gertler (1989). In Bernanke et al. (1999), MP was capable of moving real rates because of nominal rigidities. In that model, and models that follow it, Christiano et al. (2009), credit imperfections amplify the effects of the interest rate channel—through the financial accelerator. However, the effect on credit spreads is not an independent instrument, as it is here.

[8] In both models, there was a constant supply of outside money. Lucas (1980) studied a stable price equilibrium. Bewley (1983) focused on the case where money earned an interest rate financed with lump sum taxes, so the interest rate had redistributive consequences because it was funded with lump sum transfers. Ljungqvist and Sargent (2012, Chapter 18) describes how policies in Bewley (1983) models are akin to changes in borrowing limits in economies with pure credit. Lippi et al. (2015) introduce aggregate shocks into a pure currency economy, and study the optimal helicopter drops.

course, has a tradition in heterogeneous agent models (see the early work of Huggett, 1993; Aiyagari, 1994), but the literature evolved abstracting away from its initial interest in MP.

A recent generation of works has introduced nominal rigidities into heterogeneous agent models. To replicate the credit crunch of 2008, Guerrieri and Lorenzoni (2017) studies the tightening of borrowing limits in a Bewley economy with nominal rigidities.[9] These models are appealing because, as an artifact of heterogeneity, MP responses depend on the distribution of wealth and borrowing constraints. Auclert (2016) decomposes the response to policy changes into different forces that appear in that class of models. Kaplan et al. (2016) introduce illiquid assets, which produce high-income elasticities among rich agents, something that changes the nature of propagation in the new-Keynesian model.[10] In that generation of works, MP operates exclusively through the interest rate channel of the new-Keynesian model. Instead, here MP operates through the credit channel by affecting spreads.

Another set of recent works in the *money and prices* tradition, allows for credit in models where money plays a transactions role. When credit (inside money) is an imperfect substitute for outside money, the inflation-tax channel spills over to the supply of credit (see for example Berentsen et al., 2007; Williamson, 2012; Gu et al., 2015). Rocheteau et al. (2016) bring the insights of money-search transactions into a heterogeneous agent environment. The model here abstracts from the inflation-tax channels, but can naturally be adapted to feature transactions, following the methodology in Rocheteau et al. (2016).

By introducing long-term debt, another set of works, Gomes et al. (2016) for example, recognizes that MP affects the distribution of wealth through debt deflation. Nuno and Thomas (2017) take that insight to a heterogeneous agent environment and study optimal MP in a heterogeneous agent environment with nominal rigidities and possible debt deflation.

The credit channel in this paper is not new. The implementation is inherited from Bianchi and Bigio (2017a). That paper articulates a notion of the credit channel and how MP functions through corridor rates. In contrast to this paper, that paper presents a rich description of bank decisions and studies shocks that impact the interbank market, whereas the nonfinancial side is static. In that paper, any dynamic effects of MP follow from the evolution of bank net worth. Here, the banking side is simplified, but the dynamics depend on the evolution of household wealth. Piazzesi and Schneider (2016) also feature a similar implementation of MP. The focus of that paper is on the connection between interbank settlements and asset prices. Our model also shares common elements with and Brunnermeier and Sannikov (2012). In Brunnermeier and Sannikov (2012), agents face undiversified

[9] Following up on that work, McKay et al. (2015) compare the effects of forward-guidance policies in representative agent new-Keynesian models and incomplete markets economies.

[10] Greenwald (2016) and Wong (2016) study interest rate sensitivities to mortgage refinancing options.

investment risk, so a demand for currency emerges due to market incompleteness.[11]

The focus on incomplete market economies leaves room for normative analysis. The methodology employed in the normative study here follows directly from Nuno and Thomas (2017), which together with Bhandari et al. (2019), are the first papers to study optimal MP under incomplete markets. In both works, MP balances aggregate demand stabilization with insurance considerations. Instead, here the problem is to design the optimal management of the credit channel, weighing financial stability with insurance considerations. Seeing financial stability as a crucial element of MP is discussed formally in Stein (2012), for example. The normative message, that MP should actively target credit spreads, is controversial. Curdia and Woodford (2016) and Arce et al. (2019), for example, study whether the control over spreads is a useful tool in economies with nominal rigidities. Their answer is no, and that suggests that there are no costs from switching to a floor system. Instead, we take the sides of Stein (2012) and Kashyap and Stein (2012), and the control of spreads is crucial for financial stability. A corridor system is a way to achieve this stability, and moving to a floor system is a mistake.

Organization. The organization is as follows. Section 2 lays out the core model. Section 3 describes the determination of credit, interest and prices and the implementation of MP. Section 4 presents a study on MP regimes. Section 5 studies the optimal use of spreads. Section 6 concludes.

2 Environment

2.1 From Policy Spreads to Credit Spreads

In the model that follows, we embed financial intermediation (by banks) in an environment where money holdings, prices, and rates are determined in general equilibrium. In this introductory section, we present the banking block. We derive a simple formula that maps a MP corridor spread into a real intermediation spread for given monetary aggregates. Later, we show how real spreads determine monetary aggregates, and thus, how the CB has the ability to control spreads.

Banks. There is free entry and perfect competition among banks. We consider the static portfolio decision of a bank within an interval of time Δ—below, we take the limit of $\Delta \to 0$ to embed the banking block to the general equilibrium. Banks are owned by households. Because there are no aggregate shocks during the Δ interval, the bank's objective is to maximize static expected profits. Competition guarantees zero expected bank profits.

[11] Other related work includes Silva (2016), that focuses on open market operations and the effects of expected inflation. In Buera and Nicolinni (2016), the identity of borrowers and lenders is determined by a threshold interest rate. Furthermore, there is an explicit role for outside money because a transactions instruments and MP have real effects because they affect the stock of risk-free bonds which, in turn, affects the threshold identity of borrowers and lenders.

At the start of the Δ interval, banks choose their supply nominal deposits, a, nominal loans, l, and reserve holdings, m. The aggregate supply of deposits and loans, and holdings of reserves are denoted by A^b, L^b, and M^b, respectively. Deposits, loans, and reserves earn corresponding rates i^a, i^l, and i^m. Whereas the loan and deposit rates are equilibrium objects, i^m is a policy instrument.

After the portfolio decision is made, banks face random payment shocks, as in Bianchi and Bigio (2017a); Piazzesi and Schneider (2016). In particular, within the interval, payment shocks take one of two values, $\omega \in \{-\delta, +\delta\}$. Each possible value occurs with equal probability and is i.i.d across banks. If $\omega = \delta$, a bank receives δa deposits and is credited δa reserves from other banks. If $\omega = -\delta$, the bank transfers δa deposits and δa is debited to other banks. Naturally, if a bank receives a deposit, it absorbs the liability from another bank. If it loses a deposit, another bank absorbs its liability. As a result of the transfer of liabilities, assets need to be transferred to settle the transaction. A key assumption is that within the Δ time interval, loans are illiquid in the sense that they must stay with banks. Therefore, net deposit flows must be settled with reserves, which are cleared at the CB.

Upon the payment shock to a bank, the net reserve balance of a bank at the CB:

$$b = m + \min\{\omega, 0\}\, a.$$

That is, if the bank suffers a withdrawal, its balance at the CB is reduced. If the bank experiences an inflow of deposits, its overnight balance is unchanged, although the balance will increase the next day, after the position settles. Notice that deposits never leave the banking system, but a bank that receives a deposit inflow cannot lend the reserves it is owed.

Since ω is random, the reserve balance is not entirely under the control of a bank. For that reason, it is possible that the bank ends with a negative balance, $b < 0$, provided a bank starts with insufficient reserves. A bank with a negative balance must close this negative position, either by borrowing reserves from banks with a surplus or from the CB. Figure A.1 in the Appendix presents the corresponding T-accounts for the scenarios that can emerge within the Δ interval.

Interbank Market. After reserve positions are determined, an interbank market opens and banks borrow and lend reserves to each other. For a balance b, a fraction of those balances, are lent (or borrowed, if negative) in the interbank market. In particular, if a bank has a surplus b, it lends the fraction ψ^+ to other banks and, hence, $b - \psi^+ b$ remains idle in a CB account. If the bank has a deficit, $-b$, it borrows only the fraction ψ^- from other banks, and the remainder deficit $-(b - \psi^- b)$ is borrowed from the CB at a discount window rate i^{dw}. The discount rate is also a policy choice. By convention, borrowed reserves from the CB earn the interest on reserves i^m. Thus, the effective borrowing cost is the policy spread $\iota \equiv i^{dw} - i^m$. The trading probabilities $\{\psi^+, \psi^-\}$ are meant to capture trading frictions in the interbank market.

Integrating b across banks yields expressions for the aggregate surplus and the aggregate deficit

balances:

$$B^- \equiv \frac{1}{2}\max\left\{\delta A^b - M^b, 0\right\} \quad \text{and} \quad B^+ \equiv \frac{1}{2}\left(M^b + \max\left\{M^b - \delta A^b, 0\right\}\right).$$

Clearing in the interbank market requires that the total amount of reserve balances lent is equal to the amount borrowed,

$$\psi^- B^- = \psi^+ B^+. \tag{1}$$

Trading frictions, a well-documented empirical feature (see Ashcraft and Duffie, 2007; Afonso and Lagos, 2014), are key in the model to have a pass-through from policy to credit spreads. There are many ways to induce trading frictions. Here, we assume that the interbank market is an over-the-counter (OTC) market in the spirit of Afonso and Lagos (2015), but we adopt the formulation in Bianchi and Bigio (2017b) that renders analytic expressions. The interbank market works as follows: The market operates in a sequence of n trading rounds. Given the initial positions $\left\{B_0^-, B_0^+\right\} \equiv \{B^-, B^+\}$, surplus and deficit positions are matched randomly. When a match is formed between two banks, they agree on an interbank market rate for the transaction. The remaining of surplus and deficit positions define an new balance, $\left\{B_1^-, B_1^+\right\}$. New matches are formed, and a new interbank market rate emerges. The process is repeated n times, defining a sequence $\left\{B_j^-, B_j^+\right\}_{j\in 1:n}$ until a final round is reached. Whatever deficit remains is then borrowed from the CB at a cost given by ι.

The interbank market rate at a given trading round is determined by a bargaining problem in which banks take into consideration the matching probabilities and trading terms of future rounds. This produces an endogenous average interbank rate, $\bar{i}^f$. Given trading probabilities, the policy rates and the average rate $\bar{i}^f$, the average rates earned on negative and positive positions are respectively:

$$\chi^- = \psi^-\left(\bar{i}^f - i^m\right) + (1-\psi^-)\cdot\iota, \text{ and } \chi^+ = \psi^+\left(\bar{i}^f - i^m\right).$$

Banks take into account these costs and benefits when forming their portfolios. To express $\{\chi^-, \chi^+\}$, Bianchi and Bigio (2017b) assume that matches are formed on a per-position basis and according to a Leontief matching technology, $\frac{\lambda}{n}\min\left\{B_j^-, B_j^+\right\}$, where λ captures the trading efficiency. Let $\theta = B^-/B^+ \leq 1$ define an initial interbank "market tightness." In the limit $n \to \infty$, trading probabilities across all trading rounds, $\{\psi^+, \psi^-\}$, converge to $\psi^+(\theta) = \theta(1-\exp(-\lambda))$ and $\psi^-(\theta) = 1-\exp(-\lambda)$, two expressions consistent with market clearing. Then, the average interbank market rate $\bar{i}^f$ that results when both bargaining weights are equal is

$$\bar{i}^f(\theta, i^m) \equiv i^m + \iota\cdot\frac{\left((\theta+(1-\theta)\exp(\lambda))^{1/2}-1\right)}{(1-\theta)(\exp(\lambda)-1)}. \tag{2}$$

The corresponding expressions for the average cost functions are:

$$\begin{aligned}\chi^{+}(\theta) &= \iota \cdot \frac{\left(\theta\left(\theta+(1-\theta)\exp(\lambda)\right)^{1/2}-\theta\right)}{(1-\theta)\exp(\lambda)} \text{ and,} \\ \chi^{-}(\theta) &= \iota \cdot \frac{\left(\left(\theta+(1-\theta)\exp(\lambda)\right)^{1/2}-\theta\right)}{(1-\theta)\exp(\lambda)}.\end{aligned} \tag{3}$$

These coefficients are independent of i^m and only depend on the total gains from trade, $\iota = i^{dw} - i^m$. Of course, i^m affects the direct return of holding reserves. If the CB has the ability to control χ, it will have control over credit spreads.

The Bank Problem. We turn to the banks optimal portfolio choice. The average benefit (cost) of an excess (deficit) reserve balance, b, is:

$$\chi(b;\theta) = \begin{cases}\chi^{-}(\theta)\, b & \text{if } b \leq 0 \\ \chi^{+}(\theta)\, b & \text{if } b > 0\end{cases}. \tag{4}$$

We label χ the *liquidity yield* function. With this function, we are ready to present the bank's problem:

Problem 1 *[Bank's Problem] A bank maximizes its instantaneous expected profits:*

$$\Pi^b = \max_{\{l,m,a\}\in\mathbb{R}^3_+} i^l \cdot l + i^m \cdot m - i^a \cdot a + \mathbb{E}\left[\chi\left(b;\theta,\iota\right)\right]$$

subject to the budget constraint $l + m = a$ where the distribution of reserve balances is:

$$b\left(a,m\right) = \begin{cases} m \text{ with probability } 1/2 \\ m - \delta \cdot a \text{ with probability } 1/2\end{cases}.$$

At the individual level, the bank objective is piece-wise linear and, in particular, linear along a ray in the $\{m,a\}$-space. As in any model with linear firms, banks must earn zero (expected) profits in equilibrium, otherwise they would make infinity profits. Furthermore, at the individual level, banks will be indifferent among different portfolios, within a cone of in the $\{m,a\}$-space. However, at the aggregate level, the ratio of reserves to deposits will pin down the $\left\{i^l, i^m\right\}$. This feature is similar to what occurs with competitive firms that operate a Cobb-Douglas production technology with two inputs—whereas firms earn zero profits and the individual scale is indeterminate, the ratio of inputs pins down relative prices.

Equilibrium Credit Spreads. Next, we explain how a ratio of monetary aggregates determines the equilibrium loan and deposit rates. To that end, we define the aggregate *liquidity ratio* as $\Lambda \equiv$

M^b/A^b, which corresponds to the inverse of the so called "money multiplier."[12] The interbank market tightness can be expressed in terms of this ratio:

$$\theta(\Lambda) \equiv \max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}. \tag{5}$$

The tightness θ is decreasing in the liquidity ratio because with more liquidity, there is less need to borrow. The tightness decreases with Λ, and satisfies $\lim_{\Lambda\to 0}\theta = \infty$ and $\theta = 0$ for any $\Lambda \geq \delta$. If we substitute (5) into (4), we can express χ as a function of the policy corridor, ι, and the liquidity ratio, Λ, and do not depend on the level of $\{M^b, A^b\}$. Then, the linearity of the bank's problem, coupled with a free-entry condition, yield corresponding equilibrium nominal rates and a real spread:

Proposition 1 *[Nominal Rates and Real Spread] Consider an aggregate liquidity ratio Λ . Then, for given $\{\Lambda, i^m, \iota\}$, any equilibrium with finite loans and deposits must feature the following loans and deposit rates:*

$$i^l \equiv i^m + \underbrace{\frac{1}{2}(\chi^+ + \chi^-)}_{\text{liquidity value of reserves}} \tag{6}$$

$$i^a \equiv i^m + \underbrace{\frac{1}{2}(\chi^+ + \chi^-)}_{\text{liquidity value of reserves}} - \underbrace{\frac{\delta}{2}\chi^-}_{\text{deposit liquidity risk}}. \tag{7}$$

Furthermore, if $\Lambda \geq \delta$, then, $i^l = i^a = i^m$. In all cases, banks earn zero expected profits.

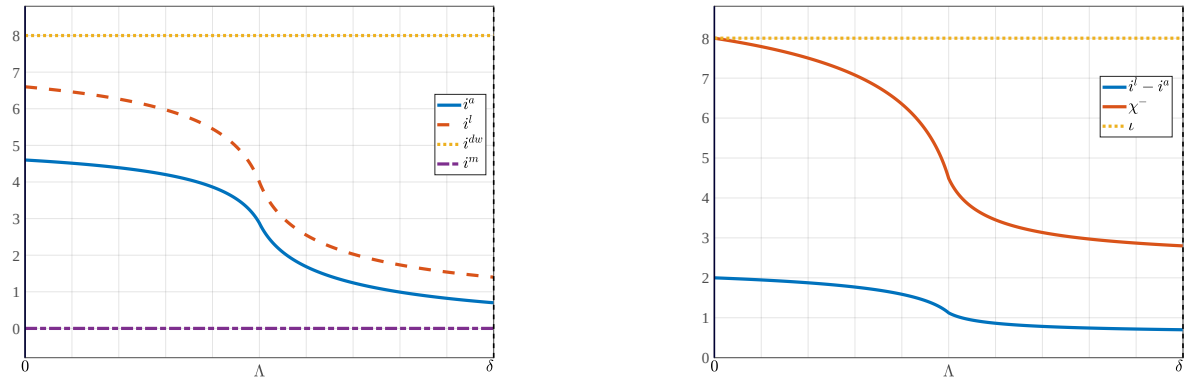
Proposition 1 establishes that the interest on reserves is a base rate for both the nominal borrowing and lending rates. Both rates carry a different liquidity premium relative to the rate on reserves. To understand this aspect, consider first the liquidity premium of loans. Loans earn a premium over reserves because, on the margin, an additional reserve earns χ^+ if the bank ends in surplus or spares the bank χ^- if the bank ends in deficit—each scenario occurs with equal probability. The deposit liquidity premium reflects that an additional deposit will increase expected costs by $\delta \cdot \chi^-$ if the bank ends with a negative balance. Thus, the deposit premium is the sum of the expected marginal increase in earnings from an additional reserve minus the expected marginal increase in payments of an additional deposit.

The loan deposit spread, a key object for the real sector, directly follows from subtracting the deposit rate from the loans rate. The equilibrium credit spread, $i^l - i^a$, is given by,

$$i^l - i^a = \frac{\delta}{2}\chi^-. \tag{8}$$

Now, the spread between two nominal rates is a real object, and thus affect household decisions,

[12] We prefer to avoid the terminology of money multiplier employed in the textbook description of the creation of deposits. According to that arithmetic, the money multiplier equals inverse of the ratio of reserve requirements, an object that is not present here.



(a) Equilibrium nominal rate given ι and Λ

(b) Equilibrium spread given ι and Λ

Figure 1: Interest Rates and Spread as Functions of Λ

Note: Panel (a) plots the nominal deposit, loan, average interbank rate, and policy rates as functions of liquidity yield and spread as functions of liquidity ratio Λ. Panel (b) shows the components of the liquidity yield and the equilibrium spread. The figure is constructed using parameters from the calibration presented in section 4.

regardless of the inflation rate. This credit spread is positive whenever the liquidity ratio is below the amount needed to satisfy the clearing of deficit banks $\Lambda < \delta$, and decreasing with the liquidity ratio. Therefore, if the CB can influence that ratio, it will influence real activity. Figure 1 depicts the formulas in Proposition 1 for nominal rates and the spread as functions of Λ. The left panel plots $\left\{i^l, \bar{i}^f, i^a\right\}$ as functions of (6) and (7) for fixed policy rates $\{\iota, i^m\}$. Both rates lie in between i^m and i^{dw}.[13] We also see how the credit spread decreases with the liquidity ratio.

The next section embeds bank intermediation into the incomplete markets economy, in the spirit of the early monetary model of Bewley (1983). Before we proceed, we discuss the assumptions encountered so far.

Digression: on discount loans and payment shocks. The discount window rate and the size of payment shocks stand in for features missing from the model. In practice, the cost of reserve shortages can be much larger than the discount window rate set by the CB. One reason for this is that discount window loans require high quality collateral. If collateral is scarce and a bank cannot close its position, the bank that cannot close a negative balance can be intervened, (for a related bank model with collateralized discount loans see De Fiore et al., 2018). Another issue is that discount window loans can bear a stigma (as in Ennis and Weinberg, 2013). This is because discount window loans are uncollateralized in the model and the discount window rate may be too low compared to the actual cost of a reserve shortage. For this reason, the discount window rate in the model must be treated as a much larger cost than the discount rate seen in the data.

Another point worth discussing is that payment shocks are i.i.d. In the data, payment shocks are likely to be persistent. To precisely capture the cost of withdrawals, we must increase the size of

[13] Credit risk or illiquidity is enough to produce rates above those bands.

shocks to compensate for the lack of persistence in the withdrawal shock. Adding persistence makes the model more realistic, but this comes at the expense of tractability.

2.2 General Equilibrium

We now embed intermediation into the general equilibrium model. We take a continuous time limit of the bank's problem. Within a Δ time interval, average profits are $\Delta \cdot \pi^b$—all rates are scaled by Δ and the objective is linear. Since bank policy functions are independent of Δ, the equilibrium rates of Proposition 1 also scale with Δ, even as $\Delta \to 0$. Next, intermediation, into the continuous-time limit of the general equilibrium. To do so, we work with a $\Delta \to 0$ limit.[14]

The non-financial sector of the economy features a measure-one continuum of heterogeneous households. From their perspective, time is indexed by some $t \in [0, \infty)$. The price of the good in terms of money is P_t. Banks intermediate between borrower and lender households, but since they make zero profits, they are simple passthrough entities. The CB determines the policy corridor rates, conducts open market operations, and makes/collects (lump sum) transfers/taxes to/from households. Households attempt to smooth idiosyncratic income shocks, via the insurance provided by the intermediation sector.

Notation. Individual-level variables are denoted with lowercase letters. Aggregate nominal state variables are denoted with capital letters. Aggregate real variables are written in capital calligraphic font. For example, a_t^h will denote nominal household deposits, A_t^h the aggregate level of deposits, and $\mathcal{A}_t^h$ real household deposits.

Households. Households face a consumption-saving problem. Household preferences are described by:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right]$$

where $U\left(c_t\right) \equiv \left(c_t^{1-\gamma} - 1\right) / \left(1-\gamma\right)$ is their instantaneous utility.
Households receive a flow of real income given by:

$$dw_t = w\left(z\right) dt.$$

Income is the sum of monetary transfers T_t (discussed below) and labor income. Labor income depends the employment status $z \in \{e, u\}$. If $z = e$, the household is employed and if $z = u$, the

[14] Note that the balance by the end of a time interval b_t, is a random variable. If we were to track b_t as a function of time, this stochastic process would not be well defined—the sum of coin tosses in continuous time is not well defined. However, treating $b_{t+\Delta}$ as the single realization of a random variable in a single instance is a perfectly well defined object.

household is unemployed. The income of the employed and unemployed are related via

$$w(e) = \left(1 - \tau^l\right) + T_t,\ w(u) = b + T_t.$$

where b is an exogenous unemployment benefit that measures a degree of labor market insurance and τ^l is a labor tax used to finance the unemployment benefit. In the expression, we are normalizing the real wage to one. The unemployment benefit b is important to control the degree of insurance in the economy and endow the unemployed with some income.

Households transition from employment to unemployment states according to an instantaneous transition probability:

$$\Gamma_t \equiv \begin{bmatrix} \Gamma_t^{eu} \\ \Gamma_t^{eu} \end{bmatrix} = \begin{bmatrix} \nu^{eu} + (\phi_t)^+ \\ \nu^{ue} - (\phi_t)^- \end{bmatrix}. \tag{9}$$

where $\{\nu^{ue}, \nu^{eu}\}$ are exogenous (natural) transition rates. The term, ϕ_t is an endogenous employment-unemployment adjustment rate that occurs do to nominal wage stickiness. Namely, ϕ_t is positive when the rigidity constraint is binding and there is an excess supply of final goods if $\phi_t = 0$. In turn, ϕ_t is negative when the rigidity constraint is binding and there is an excess demand of final goods under $\phi_t = 0$. The transition matrix, $\Gamma_t^{zz'}$ measures the endogenous transition rate from state z to state z', where $z \neq z'$.

All financial assets are nominal. Although all claims are nominal, the individual state variable is, s_t, a stock of real financial claims. Households store wealth in bank deposits, a_t^h, or currency, m_t^h, and borrow loans against banks, l_t^h. By convention, $\left\{a_t^h, m_t^h, l_t^h\right\} \geq 0$. The real rates of return on deposits and liabilities are $r_t^a \equiv i^a - \dot{P}_t/P_t$ and $r_t^l \equiv i^l - \dot{P}_t/P_t$. Currency doesn't earn nominal interest, and thus, its real return is $-\dot{P}_t/P_t$. The law of motion of real wealth follows:

$$ds_t = \left(r_t^a \frac{a_t^h}{P_t} - \frac{\dot{P}_t}{P_t}\frac{m_t^h}{P_t} - r_t^l \frac{l_t^h}{P_t} - c_t\right) dt + dw_t \tag{10}$$

and the balance-sheet identity:

$$\left(a_t^h + m_t^h\right)/P_t = s_t + l_t^h/P_t.$$

From a household's perspective, there is no distinction between holding deposits or currency beyond their rates of return. Hence, currency is only held when the nominal deposit rate is zero, and both assets yield the same return. Importantly, currency is introduced into the model to articulate a DZLB as an implementation constraint. Another observation is that households will never hold deposits and loans, if there's a positive spread between them. Combining these insights, (10) can be written

more succinctly as:

$$ds_t = (r_t(s)\, s - c_t)\, dt + dw_t \text{ where } r_t(s) \equiv \begin{cases} r_t^a \text{ if } s_t > 0 \\ r_t^l \text{ if } s_t \leq 0 \end{cases}. \tag{11}$$

Another important assumption is that employment risk cannot be diversified due to incomplete markets. In particular, households can borrow, but with some limitations. Concretely, credit is limited by a *debt limit* $\bar{s} \leq 0$. This limit determines an absolute lower bound on real debt, $s_t \geq \bar{s}$ where $\bar{s} \leq 0$ is exogenous and constant. Technically, this means that at $s = \bar{s}$, it must be that $ds_t \geq 0$. With these constraints, the corresponding household Hamilton-Jacobi-Bellman (HJB) equation is:

Problem 2 *[Household's Problem] The household's value and policy functions are the solutions to:*

$$\rho V(z,s,t) = \max_{\{c\}} U(c) + V'(z,s,t)\left[r_t s - c + y(z) + T_t\right] + \Gamma_t^{zz'} \cdot \left[V\left(z',s,t\right) - V(z,s,t)\right] + \frac{\partial V(z,s,t)}{\partial t}$$

and $\dot{s} \geq 0$ *at* $s = \bar{s}$.

We define by the drift of wealth by $\mu(z,s,t) \equiv r_t s - c + y(z) + T_t$.

Inflation. The price level moves according to a

$$\dot{\pi}(t) = \rho\left(\pi(t) - \pi_{ss}\right) - \kappa\left(u_{ss} - u_t\right). \tag{12}$$

This is a classic forward-looking Phillips curve (NS), where we use the unemployment rate above/below the natural rate u_{ss}. In the expression, π_{ss} is a long-run expected inflation-target implemented by the CB interest-rate policy.

Solving the equation Forward, delivers the following integral solution for inflation:

$$\pi(t) = \pi_{ss} + \kappa \int_0^\infty \exp(-\rho s)\left(u_{ss} - u_{t+s}\right) ds.$$

Importantly, $\pi(t)$ is not pre-determined as it depends on path of future unemployment. Inflation is boosted at intensity κ, as unemployment falls below steady state. When unemployment is below steady state, the economy experience wage pressure. In that case, wages tend to increase. Similarly, the economy features deflation as the unemployment rate rises above steady state. When $\kappa \mapsto \infty$, the economy approaches a flexible price equilibrium, and as $\kappa \mapsto 0$, it converges to a constant price equilibrium.

Distribution of Wealth and Employment Status. The mass of households sums to one. Among them, the fraction e is active in the work force, and the fraction u is unemployed. The mass of

unemployed evolves according to the law of motion derived from the transition probabilities:

$$\dot{u} = \left[\nu^{eu} + (\phi_t)^+\right] \cdot (1-u) - \left[\nu^{ue} - (\phi_t)^-\right] \cdot u.$$

At each instant, there's a distribution $f(z,s,t)$ of real financial wealth across households given their employment status z. The law of motion of this distribution satisfies a Kolmogorov-Forward Equation (KFE): $m^h(s,t)$ be the solutions to the household's problem. The KFE of f is,

$$\begin{aligned}
\frac{\partial}{\partial t} f(e,s,t) &= -\frac{\partial}{\partial s}\left[\mu(e,s,t) f(e,s,t)\right] - \Gamma_t^{eu} \cdot f(e,s,t) + \Gamma_t^{ue} \cdot f(u,s,t), \text{ and} \\
\frac{\partial}{\partial t} f(u,s,t) &= -\frac{\partial}{\partial s}\left[\mu(u,s,t) f(u,s,t)\right] - \Gamma_t^{ue} \cdot f(u,s,t) + \Gamma_t^{eu} \cdot f(e,s,t).
\end{aligned} \tag{13}$$

Central Bank. The CB maintains zero equity every period. Thus, its assets and liabilities are equal in every period, $L_t^f = M_t$. CB holds as assets L_t^f, and issues liabilities, that represent the monetary base, M_t. The monetary base is divided into reserves M_t^b and currency, $M0_t$. In the paper, we assume that banks can't hold currency due to regulation, taxation, or physical costs. An open-market operation (or a reverse open-market operation) is a simultaneous increase (or decrease) in M_t and L_t^f. Because of interest rate differentials, and its discount-window loans, the CB generates operational profits which it distributes to the central government. In addition to these operations, the CB also sets the discount window rate i_t^{dw}, the rate at which banks can borrow reserves and the interest on reserves, i_t^m, that we presented earlier.[15] In principle, we can think of $\{i_t^m, i_t^{dw}\}$ as independent instruments. However, for the rest of the paper, we assume that the policy corridor spread $\iota = i_t^{dw} - i_t^m$ is a constant.

The operational profits of the CB are:

$$\Pi_t^{CB} = i_t^l L_t^f - i_t^m (M_t - M0_t) + \iota_t \left(1 - \psi_t^-\right) B_t^-. \tag{14}$$

The CB earns i_t^l on L_t^f, and pays i_t^m on the portion of the money supply held as reserves, hence it earns an interest rate differential. The third term, $\iota_t \left(1 - \psi_t^-\right) B_t^-$, is the income earned from discount window loans.

The CB operational income plus the surplus or deficit of from social insurance, $e_t \tau^l - u_t b$, are the total revenues of the government, which we assume are distributed lump-sum:

$$P_t T_t dt = \Pi_t^{CB} dt + P_t \left(\tau^l e_t - b u_t\right).$$

[15] The CB faces a solvency restriction, $i_t^{dw} - i_t^m \geq 0$, and also $i^{dw} \geq 0$. The spread $i_t^{dw} - i_t^m \geq 0$ because a negative corridor spread would enable banks to borrow from the discount window and lend back to the CB and create arbitrage profits. If $i^{dw} < 0$, banks could borrow reserves and lend reserves as currency to households swapping the currency for deposits at zero rates. This operation would produce another arbitrage for the bank.

To set the interest on reserves, the CB works with a Taylor rule that allows for both, a discretionary component that is triggered during a credit crunch, but also follows a standard Taylor rule that captures commitment for the long-run. This feature is important. Without a Taylor rule, the price-level is unstable, so we need the long-run component. At the same time, we want to capture the idea that MP responds to economic conditions. For the rest of the paper, we assume T_t adjusts to satisfy the balanced budget above (fiscal passive regime).

A Time-Varying Taylor Rule. For that, we specify the following rule:

$$i_t^m = \bar{i}_t^m + \eta_t \cdot (\pi_t - \pi_{ss}) \tag{15}$$

where η_t the time-varying response of the Taylor-rule to inflationary pressures. Importantly, $\bar{i}_t^m$ is a time-varying process that induces a long-run inflation target. We use this parameter to study the effects of changes in nominal policy rates. It is important to let η_t and $\bar{i}_t^m$ vary over time. First, by letting $\bar{i}_t^m$ change, we can analyze the effects of changes in the monetary policy rule. However, we also need to change the response of the policy rule to inflation to properly isolate the responses of variables. Thus, this formulation is flexible enough to allow for isolated changes in the interest rate rules, but in ways that the economy is permitted to transition to the one governed by a standard Taylor rule in the long-run.

Markets. Outside money is held as bank reserves or as currency. The aggregate currency stock is

$$M0_t \equiv \int_{\bar{s}}^{\infty} m_t^h(s) f(s,t)\, ds.$$

Equilibrium in the outside money market is:

$$M0_t + M_t^b = M_t. \tag{16}$$

The credit market has two sides, a deposit and a loans market. In the deposit market, households hold deposits supplied by banks. In the loans market, households obtain loans supplied by banks. The distinction between the loans and deposits is that they clear with different interest rates. The deposit market clears when:

$$A_t^b = \int_0^{\infty} a_t^h(s) f(s,t)\, ds, \tag{17}$$

where $a_t^h(s) \equiv P_t s - m_t^h(s)$, for a positive s. The left of this equation is the bank supply of deposits. The loans market clears when:

$$L_t^b + L_t^f = \int_{\bar{s}}^{0} l_t^h(s) f(s,t) ds, \tag{18}$$

where $l_t^h(s) \equiv -P_t s$ for negative s. Finally, the goods market clears when:

$$\int_{\bar{s}}^{\infty} y(u(s,t)) f(s,t) ds \equiv Y_t = e_t = C_t \equiv \int_{\bar{s}}^{\infty} c(s,t) f(s,t) ds. \tag{19}$$

Equilibrium. A price path-system is the vector function $\{P(t), i^l(t), i^a(t)\} : [0,\infty) \to \mathbb{R}^3_+$. A policy path is the vector function $\left\{L_t^f, M_t, , i_t^{dw}, i_t^m, T_t\right\} : [0,\infty) \to [0,\infty) \to \mathbb{R}^5_+$. Next, we define an equilibrium path.

Definition 1 *[Perfect Foresight Equilibrium.] Given an initial condition for the distribution of household wealth f_0, an initial price level P_0, and a policy path $\left\{\mathcal{L}_t^f, i_t^m, \iota_t, T_t\right\}$, a perfect-foresight equilibrium (PFE) is (a) a path for inflation, (b) a path for the real wealth distribution f, (c) a path of aggregate bank holdings $\{L_t^b, M_t^b, A_t^b\}_{t\geq 0}$, (e) unemployment flows, and (d) household's policy $\{c, m^h\}$ and value functions $\{V\}_{t\geq 0}$, such that:*

1. *The path of aggregate bank holdings solves the static bank's problem (1) at each t,*
2. *The household's policy rule and value functions solve the household's problem (2),*
3. *The unemployment transitions satisfy (9),*
4. *The law of motion for $f(s,t)$ is consistent with Kolmogorov-Forward equation (13),*
5. *The government's policy path satisfies the governments budget constraint (14),*
6. *All the asset markets and the goods market clear (1,16-19).*

Next, we characterize the equilibrium dynamics. A **steady state** occurs when $\frac{\partial}{\partial t} f(s,t) = 0$ and $\{r_t^a, r_t^l\}$ are constant. We use subscripts *ss* to denote variables at steady state.

Digression: Model Assumptions. The financial architecture in the model captures a fundamental feature of banking. In practice, banks issue deposits in two transactions. The first is a swap of liabilities with the non-financial sector. When banks make loans, they effectively credit borrowers with deposits, a bank liability is exchanged for a household liability. This swap is the process of inside money creation. Deposits then circulate as agents exchange deposits for goods. This circulation gives rise to the settlement positions. The second transaction is the exchange deposits (a bank liability) for currency (a government liability).

A missing element is government bonds. In practice, central banks conduct open-market operations by purchasing government bonds. Here, negative holdings of L^f are interpreted as government bonds. The implicit assumption is that bonds are as illiquid as private loans. Bianchi and Bigio (2017a) introduce government bonds that are more liquid than loans, but less so than reserves, because they cannot be used for settlements.

3 Implementation

We begin with a simplest of observations. A spread between nominal rates equals the spread between the corresponding real rates. This observation is important because households take decisions based on real rates. If the central bank can control a nominal spread, it implies it can control more than one real rate. As shown in Proposition 1, the spread is governed by the liquidity ratio, Λ. We now explain how the CB implements a desired credit spread by conducting open-market operations, and then how the spread affects the steady-state and transitions in this model.

Implementation. From equation (8), we know that the real spread Δr_t is a function of the liquidity ratio Λ_t. A natural question is to what extent does the CB control the liquidity ratio? The main result of this section is that OMO affects the liquidity ratio, unless the economy reaches a DZLB or unless the economy is satiated with reserves.

We first characterize the DZLB. The DZLB emerges because households can convert deposits into currency. Hence, although the CB can set a negative interest $i_t^m < 0$, the deposit rate is always satisfies $i_t^a \geq 0$ in equilibrium. To characterize the DZLB, we define Λ_t^{zlb} as the threshold liquidity such that for any liquidity ratio above that point, the equilibrium deposits rate, as determined by equation (7), would be negative:

$$\Lambda^{zlb}\left(i_t^m, \iota_t\right) \equiv \min\left\{\Lambda | 0 \geq i^m + \frac{1}{2}\left(\chi^+\left(\Lambda\right) + \left(1-\delta\right)\chi^-\left(\Lambda\right)\right)\right\}.$$

Because a negative deposit rate cannot occur in equilibrium, in any equilibrium $\Lambda_t \leq \Lambda^{zlb}\left(i_t^m, \iota_t\right)$. If the CB attempts to increase Λ_t, beyond Λ^{zlb}, the increment in the money supply must immediately translate into an increase in currency $M0_t$, but not in M_t^b! Thus, at the DZLB the CB loses the ability to influence spreads through an increase in OMO. Furthermore, because $\chi^- \geq \chi^+ \geq 0$, we know that Λ^{zlb} is a finite only if $i_t^m < 0$. Thus, the DZLB is relevant only when i^m is negative.

We can define *a monetary base liquidity ratio*, Λ_t^{MB} as $\Lambda_t^{MB} \equiv M_t / A_t$. Different from the liquidity ratio of banks, Λ_t, which is the relevant object to determine bank spreads, the monetary base liquidity ratio Λ_t^{MB} is defined in terms of the total monetary base, which includes reserves and currency. We

express Λ_t^{MB} in terms of the composition of the CB's balance sheet in real terms:

$$\Lambda_t^{MB} = \frac{L_t^f / P_t}{A_t / P_t} = \frac{\mathcal{L}_t^f}{\int_0^\infty s f(s,t)\, ds} \equiv \Lambda^{MB}\left(f_t, \mathcal{L}_t^f\right).$$

In addition to the DZLB regime, OMO are also irrelevant when banks are satiated with reserves and $\theta = 0$. This regime occurs when $\Lambda_t \geq \delta$, because in that case, banks have enough liquidity to cover a withdrawal. When, $i^m < 0$, we know that $\Lambda^{zlb}(i_t^m, \iota_t) < \delta$, because otherwise $\chi^- = \chi^+ = 0$, and the deposit rate would be negative. This means that the satiation regime occurs only when rates on reserves are positive. This observation, allows us to organize the effects of policy tools into a single proposition:

Proposition 2 *[Properties of Equilibrium Rates and Spreads] Consider a CB policy given by* $\left\{\mathcal{L}_t^f, i_t^m, \iota_t\right\}$.

1. *Corridor Regime: Let* $\Lambda_t < \min\left\{\delta, \Lambda^{zlb}(i_t^m, \iota_t)\right\}$, *then* $i^m < i^a < i^l < i^{dw}$ *and* $\Delta r \in (0, \iota)$. *Furthermore, the effect of policy instruments on the spread is:*

$$\frac{\partial \Delta r}{\partial \mathcal{L}_t^f} < 0 \text{ and } \frac{\partial \Delta r}{\partial i_t^m} = 0.$$

2. *Floor regime: Let* $i^m \geq 0$ *and* $\Lambda_t \geq \delta$, *then* $i^l = i^a = i^m$ *and* $\Delta r = 0$. *Furthermore, the effect of policy instruments on the spread is:*

$$\frac{\partial \Delta r}{\partial \mathcal{L}_t^f} = 0 \text{ and } \frac{\partial \Delta r}{\partial i_t^m} = 0.$$

3. *NIOR and DZLB regime: Let* $i^m < 0$ *and* $\Lambda_t = \Lambda^{zlb}(i_t^m, \iota_t)$, *then* $i^l > i^a = 0$ *and* $\Delta r > 0$. *Furthermore, the effect of policy instruments on the spread is:*

$$\frac{\partial \Delta r}{\partial \mathcal{L}_t^f} = 0 \text{ and } \frac{\partial i^l}{\partial i_t^m} = \frac{\partial \Delta r}{\partial i_t^m} < 0, \frac{\partial i^a}{\partial i_t^m} = 0.$$

Proposition 2 establishes the direction of policy effects. There are three regimes: In the first regime, $\Lambda_t < \min\left\{\delta, \Lambda^{zlb}(i_t^m, \iota_t)\right\}$ so liquidity is scarce enough to promote interbank lending. In CB jargon, this regime is a *corridor system*. It features a positive credit spread controlled via open market operations. Increases in i^m induce a parallel increase in both nominal rates, with effects on employment and inflation, but keep the spread constant. The neutrality of i^m on the spread implies that the CB can control inflation independently from the impact on spreads, which is its balance sheet policy.

When $i^m > 0$ and the liquidity ratio exceeds, $\Lambda > \delta$, banks are satiated. In that case, all nominal rates equal i^m. This regime is a *floor system*. Furthermore, OMO have no effects and therefore satisfy classic

Wallace irrelevance, (Wallace, 1981). In a floor system, the CB loses the ability to affect spreads and can only handle inflation. Appendix B presents additional figures for the case of a negative rate on reserves.

Now consider a regime with negative interest on reserves, $i_t^m < 0$. This regimes opens the possibility of a DZLB. A DZLB occurs when the liquidity ratio is above the endogenous liquidity ratio Λ_t^{zlb} that makes the deposit rate hit zero. In that region, OMO are irrelevant because any increase in CB liabilities translates into an increase in currency, not reserves. However, the spread is still positive, even though OMO have no effects. The reason is that negative rates on reserves tax deposits. Since the deposit rate is fixed at zero, banks require a higher lending rate—because deposits have an infinite price elasticity at that rate. As a result, changes in i_t^m produce a joint effect on the real spread and inflation, which is something that does not occur in a corridor system. The change of behavior of spreads at the DZLB has been documented by (Heider et al., 2019; Eggertsson et al., 2019). In different models, Brunnermeier and Koby (2019) and Ulate (2019) obtain a similar effect, but the mechanism operates through bank capital. Figure 16 in...

Two Intermediate results. Next, we discuss the steady state and how MP affects the steady state. We then discuss the effects along a transition. then discuss the effects of policies along a transition. Before that, we derive the clearing conditions of this economy.

We begin by expressing the government budget constraint in real terms. If the CB induces a real spread, and banks earn zero profits, the revenues from the spread must go somewhere. The only possibility is that the revenue from the spread goes to the operational profits on the CB. The next proposition uses this observation to relate the real fiscal transfers to the spread and the fiscal deficit:

Proposition 3 *[Real Budget Constraint] The real spread produces transfers equal to*

$$T_t = \underbrace{\Delta r_t \cdot \int_0^\infty sf\,(s,t)\,ds}_{\text{operational revenue}} + \underbrace{\tau^l e_t - bu_t}_{\text{fiscal deficit}}. \tag{20}$$

Given $\{\Delta r_t, T_t\}$, market clearing in real financial claims is consistent with an equilibrium real deposit rate r_t^a and a job separation rate ϕ_t. This real equilibrium rate is the one that solves a single clearing condition which, in turn, implies clearing in all asset markets:

Proposition 4 *[Real Wealth Clearing] Let nominal rates be given by (6) and (7), and let the liquidity ratio be given by Λ_t. Then, market clearing in real wealth,*

$$0 = \int_{\bar{s}}^\infty sf\,(s,t)\,ds \;\; \textit{for } t\in[0,\infty), \tag{21}$$

*implies market clearing in all asset markets. Furthermore, if (**??**) and the Kolmogorov-Forward equation (13) hold, then, the goods market clearing condition (19) holds.*

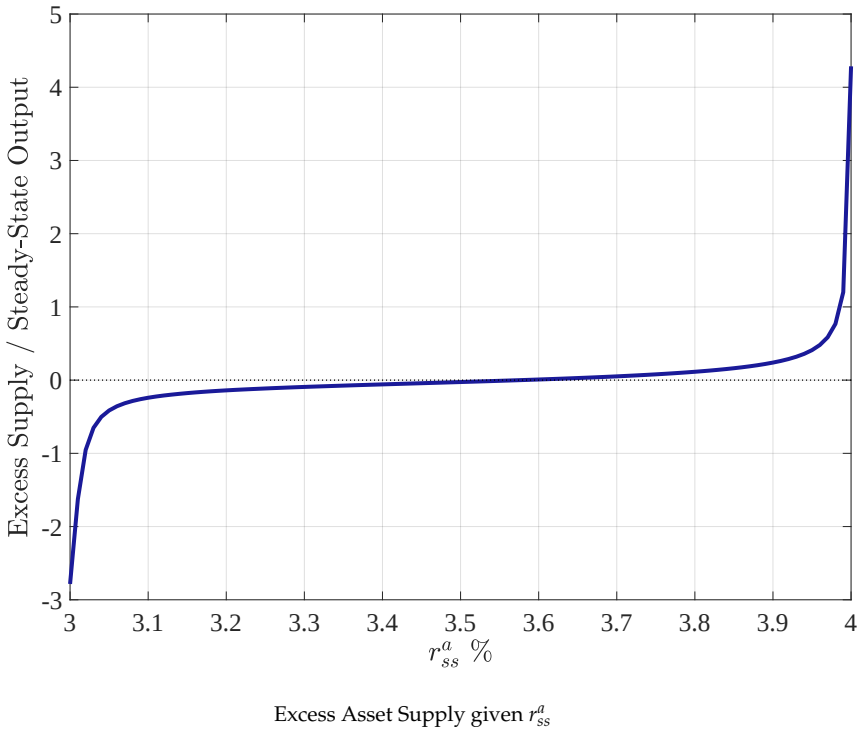


Figure 2: Excess Savings as a Function of Deposit Rate in Steady State

Note: The figure depicts the excess savings supply as a function of the real deposit rate (in steady state). Taking the real spread as given, the spread is constant. The figure is constructed using parameters from the calibration presented in Section 4.

The proposition shows that all clearing conditions are summarized by a single condition. To guarantee clearing along a transition, we must obtain a real deposit rate, r^a_t. For example, Figure 2 plots the relationship between the excess supply in real savings relative to GDP at a steady state, as a function of a real deposit rate r^a_t.

Steady State. Consider now a steady state. Let the CB have a specific target long-run target for Δr_{ss}. At steady-state, the disturbance in job-separation ϕ_t must be zero because that is the only possibility consistent with the Phillips curve that generates no acceleration in inflation. We also know that that inflation has no effects in a steady state—unlike the new-Keynesian model. In other words, money is superneutral. Thus, at steady-state, the real interest rate r^a_{ss} adjusts to solve:

$$0 = \int_{\bar{s}}^{\infty} s f_{ss}(s)\, ds.$$

Once we obtain an equilibrium r^a_{ss} in steady state which also corresponds to the real interest rate that prevails in flexible price equilibrium, we obtain the path of inflation:

$$\pi_{ss} = \pi_\infty = i^m_\infty - r^a_\infty + \frac{1}{2}\left[\chi^+_\infty + (1-\delta)\,\chi^-_\infty\right].$$

Once inflation is obtained, all nominal variables grow at the rate of inflation. Finally, to implement, Δr_{ss} we find the size of monetary base such consistent with Λ_t such that Δr_{ss} is given by (8).

Transitions. Along a transition, things work differently. In particular, π_t is pre-determined by the Phillips curve (12). Given i_t^m and a liquidity ratio, i_t^l and i_t^a are determined. Hence, r_t^l and r_t^a follow from the Fisher equation,

$$r_t^x = i_t^x - \pi_t \text{ for } x \in \{l, a\}. \tag{22}$$

To satisfy clearing in the asset market, then, the job separation rate adjusts to satisfy (21). We obtain the following:

Proposition 5 *[Implementation Conditions] Consider an equilibrium path for $\{r_t^a, \Delta r_t, f_t, \pi_t, \phi_t\}_{t\geq 0}$. To implement the equilibrium path, the CB chooses $\left\{i_t^m, \mathcal{L}_t^f\right\}$ subject to the following restrictions:*

1. *The equilibrium liquidity ratio is $\Lambda_t = \min\left\{\Lambda^{zlb}\left(i_t^m, \iota_t\right), \Lambda^{MB}\left(f_t, \mathcal{L}_t^f\right)\right\}$.*
2. *The real transfer, T_t adjusts to satisfy (20).*
3. *The real spread Δr_t is given by (8) for given Λ_t.*
4. *Given i_t^m and Λ_t the nominal rates $\left\{i_t^l, i_t^a\right\}$are given by (6-7).*
5. *Given ϕ_t, the unemployment rate u_t satisfies the law of motion (9).*
6. *Inflation is consistent with the Phillips curve, (12).*
7. *The real rates $\left\{r_t^l, r_t^a\right\}$ are consistent with Fisher's equation(22).*
8. *The distribution of real wealth, f_t , evolves according to (13) with f_0 given.*
9. *Given f_t, the job separation ϕ_t guarantees the real asset market clearing condition (21).*

Proposition 5 describes the dynamic allocations that can be induced by the CB. Allocations are affected by the CB because it controls the real spread by conducting OMO and through the effects of i_t^m.

Policy Discussion: Tools and Targets. At any point, in the current formulation, the CB here has two direct tools, $\left\{i^m, \mathcal{L}_t^f\right\}$. We observed that i^m controls inflation through the Phillips curve. We also saw that given ι a choice of $\mathcal{L}_t^f$ can produce a desired spread. To understand whether the latter are redundant assets, we must ask if the implementation of a credit spread has fiscal consequences. In the model, the smae spread can be obtained by moving the corridor spread ι, or by implementing OMO. We could be tempted to argue that these instruments have different fiscal consequences, but they don't as Proposition **??** shows:

Corollary 1 *[No Fiscal Consequence of an implementation choice] Consider two policies $\{\iota, \Lambda_t\}$ that implement the same real spread target, Δr_t. Both are consistent with the same discount window profits and, hence, produce the same fiscal revenues.*

In summary, $\left\{\iota, \mathcal{L}_t^f\right\}$ have the same effect on households through the spread, and have the same effect on fiscal revenues, both instruments are redundant.[16]

Policy Discussion: Alternative Implementations, the DZLB, and Fiscal-Monetary Interactions. It is worth discussing MP implementations used in practice (Bindseil, 2014, reviews cross-country practices.). In the model, one alternative way to the control the real spread directly through OMO while keeping ι constant, is to target an interbank market rate $\bar{i}^f$: given ι, we can find a consistent Λ that delivers $\bar{i}^f$. Because there is also a map from $\bar{i}^f$ to Δr_t, a target for the interbank rate also implements a spread.

In practice, most CBs have an explicit interbank target, but restrict they way in which they achieve it. So CBs set corridor systems with a constant corridor width ι and move Λ and target an interbank market at the middle, $\bar{i}^f = i^m + \frac{1}{2}\iota$. Other countries, keep the rate on reserves at zero but move ι, and simultaneously target $\bar{i}^f$ at a constant distance from ι. Our analysis suggests that under either system, a CB will simultaneously move spreads and inflation, perhaps inadvertently. However, in doing so, CBs lose an instrument: they can target inflation and spreads within a given mix, but not as independent targets. If CBs are open to move i^m and ι, they can reach both targets. This paper argues that targeting spreads is desirable.

The effects of policy at the zero lower bound are different from those that emerge in cash-in-advance constraints. In those models, a ZLB emerges if the CB floods the public with savings instruments so that the asset clears at negative rates. That opens the door to an unrealistic arbitrage in which the households borrow at negative rates from the CB to hold currency.[17] Here, it is important to note that while the ZLB applies to the nominal deposit rate, it does not apply to the rate on the policy instruments. The policy conclusion is that a CB that reduces i_t^m, perhaps in an attempt to increase inflation, will cause an increase in credit spreads.

The model inherits classical monetary properties in Bewley economies (Bewley, 1983; Ljungqvist and Sargent, 2012, , Chapter 18.11). First, a version of the quantity theory holds. If we fix a path for T_t, and Δr_t, then we can scale every nominal variable by a scalar and obtain the same equilibrium. Second, changes in the growth rate of nominal transfers produce an increase in steady-state inflation. If i^m increases at that same rate, the effect of the policy is neutral.[18] Third, because the economy is

[16] However, ι can be increased to achieve any spread. Instead, OMO can produce spreads within the bound $\Delta r \in \left\{\frac{\iota}{2}, \iota\right\}$.

[17] In Rognlie (2016), negative rates are possible because there are costs of holding physical currency.

[18] To illustrate, assume a policy where from t onward, the CB increases the growth rate of nominal transfers. Then inflation rate will increase at a constant rate as long as the CB keeps a real transfers constant, but the CB must also increase i_t^m and i^{dw} by the new rate of inflation. Thus, if the CB moves its policy rates accordingly, monetary policy is super neutral. If instead, the CB increases transfers but keeps the real rate constant, it effectively changes the real value of transfers and the real rate. In that case, monetary policy is not super neutral.

neutral, there is potentially a continuum of equilibrium indexed by time zero prices. By normalizing P_0, we fix the initial distribution of wealth.

4 Positive Analysis: From Instruments to Channels

This section discusses the positive analysis of the effects of MP within the three regimes described earlier. First, we discuss the regime where MP eliminates any credit spread (a floor system) achieved by satiating banks with reserves. We then move of to discuss a regime that opens spreads (a corridor system). We then move on to discuss the effects of policy at the DZLB. Table 1 presents a summary of the instruments that are operative under each regime (corresponding to Proposition 5) and the channels that they activate. At the end of this section, we discuss the effects of a credit crunch, as a prelude to the normative analysis of the following section.

	Instrument		Channel		
Regime	i_t^m	L_t^f	Fisherian	Non-Ricardian	Credit
Floor system (sec 4.1)	✓		✓		
Corridor system (sec 4.2)	✓	✓	✓	✓	✓
DZLB (sec 4.2)	✓			✓	✓

Table 1: Instruments and Transmission Channels

Calibration. At this stage, we must proceed with numerical examples. Next, we present a calibration to produce the computations. The paper has many missing elements that would allow for a proper quantitative analysis. The spirit, nonetheless, is to provide a quantitative sense of the strength of the different transmission mechanisms. The calibration, is also a guide to where the model needs realism. The calibration is summarized in Table 2 and inspired by the US economy. Risk aversion, which coincides with the inverse inter-temporal elasticity, γ, is set to 2. The time discount, ρ, is set to 4%, which yields a real steady-state deposit rate of approximately 1.0%.

The unemployment benefit is set to 0.41% matching the unemployment insurance replacment. The labor tax τ^l is set to 0.3, the average labor-income tax. The coefficient of the Taylor rule η is set to 1.5, a standard value. In turn, κ in the Phillips curve is set to 0.1, as in Nakamura and Steinsson xxx.

The interbank-market efficiency, λ, is set to 2.1. This number is directly taken from Bianchi and Bigio (2017a), who calibrate it to match the size of discount window loans. The rate on reserves is set to $i_{ss}^m = r_{ss}^a$, so steady state inflation is zero, which is a normalization. The discount window rate is set to produce a steady-state spread of 2%, for the exercises where the spread is open. The required spread between the discount-window rate and the rate on reserves is much higher than in the data, but as we argued above, this is a stand-in for missing elements such as collateral and stigma (De Fiore et al., 2018). The average shock δ is set to produce the same market tightness in the interbank-market as in Bianchi and Bigio (2017a).

Parameter	Value	Description	Target/Reference
γ	2	Risk aversion	standard
ρ	0.04	time discount	1.0% real risk-free rate
ν^{eu}	0.4	Job separation rate	Shimer (2005)
ν^{ue}	5.4	Job finding rate	Shimer (2005)
κ	0.1	Phillips curve parameter	Nakamura Steinsson (2020)
b	0.41	Unemployment benefit	UI Replacement
τ^l	0.3	Proportional labor tax rate	U.S. average labor-income tax (OECD)
η	1.5	Taylor rule parameter	Taylor (1993)
Δr	2%	Credit spread	
$\bar{s}$	$-12w(u)$	Credit limit	

Table 2: Parameter Values

Note: The table lists the calibrated values of parameters and the corresponding reference/target of calibration.

The debt limit $\bar{s}$ is set to $-12w(u)$ to produces a debt-to-income ratio of 12 for the poorest households, a number in line with the literature. The borrowing limit $\tilde{s}$ is set to $\bar{s}$ so it is active only when we study a credit crunch.

A standard way to calibrate the unemployment-to-employment and empoloyment-to-unemployment rates **is to directly use the transition rates as in Shimer (2005) which estimates values for $\{\nu^{eu}, \nu^{ue}\}$ of $\{0.4, 5.4\}$. However, we want our calibration to acknowledge that there is much more labor income risk and, actually, consumption income risk in the data. Thus, we focus on values for $\{\nu^{eu}, \nu^{ue}\}$ that allow us to generate a good fit for the fraction of agents in debt, and the fraction of agents at their debt limit.**

Steady-state Moments. To get a sense of quantitative fit, we report steady-state moments in Table 3. The model produces a 0.0% share of agents at their debt limit and a 45.9% share that hold positive debt. The CB's operational profits are 5.6% of output. In the US, the transfers of the Federal Reserve to the Federal Government are similar to corporate tax revenues, about 1.8% of GDP. Since the model does not have operational costs for banks nor the CB, this figure, which is three times as high as in the data, is reasonable. The interest expense on the CB's position is 1.4% of GDP. Finally, we report levels of wealth over GDP measured as wealth divided by per-capita income, at different quantiles. The model misses the return shocks needed to produce the concentration ratios at the top, but does a fair job at the bottom of the distribution. Since, as we show, most of the dynamics stem from the behavior of the poor, missing the wealth concentration at the top should not have an important effect on the quantitative responses.

Logistic Shocks. All of the shocks in this section will follow a logistic path.

[TBA]

Moment	Value
Fraction of households at debt limit	0.0%
Fraction of households in debt	45.9%
CB operational revenue / GDP	5.6%?
CB interest rate expense / GDP	1.4%?

Table 3: Additional Moments (Not-Targeted)

Note: The table reports the untargetted moments of the calibrated model.

4.1 A Floor System and the Fisherian Channel

We begin with a policy that neutralizes the spread, but nonetheless can attain a nominal rate target, i.e., a floor system. We begin explaining the effects in a flexible price economy.

Flexible Prices and Steady State. Consider the flexible price version of the economy. We have the following corollary of Proposition 5:

Corollary 2 *[Floor System] Let the CB set $i_t^m \geq 0$ and $\Lambda_t \geq \delta$. Then, the spread is zero, $\Delta r_t = 0$, and the evolution of $\{r_t^a, f(s,t)\}$ is unaffected by policy. Inflation is controlled by i_t^m as given by (22).*

Corollary 2 is a special case of Proposition 5. Under flexible prices, if the CB satiates banks with reserves or eliminates the policy corridor, monetary policy is operates through the Fisher equation. That is, the CB controls inflation as it effectively resets the unit of account every period, simply increases the money supply (and the private sector increases it nominal liabilities) every period. The result showcases that the CB can control nominal rates, without the need of OMO.[19] The ability to control inflation with a single instrument connects with three, well-traveled transmission mechanisms: the New-Keynesian channel, the inflation-tax, and debt deflation, which we reviewed above.[20] Naturally, in the flexible price economy, changes in the rate of reserves are entirely neutral, but this is not the case once we have price rigidity.

Rigid prices. Next, we return to the effects of changes in i_t^m under price rigidity. We reproduce similar effects to new-Keynesian models with incomplete markets, (Guerrieri and Lorenzoni, 2017; Kaplan et al., 2016; Auclert, 2016). In Figure 4, we consider an the exercise that deviates from a constant Taylor rule by inducing a change in the the discretionary component of the Taylor rule $\bar{i}_t^m$, (15). In this experiment, the central bank chooses the initial value $\bar{i}_0^m$ of the discretionary component

[19] Without a policy corridor, the CB controls inflation even if there are no reserves. Woodford (2001) advocates for this policy and labels it a Wicksellian doctrine.

[20] One can also incorporate sporadic currency transactions as in Rocheteau et al. (2016), that would produce an interesting interaction between currency holdings, the distribution of wealth, and productive efficiency that would be affected by the inflation tax channel. One can also lengthen loan terms so that unexpected changes in i_t^m are not neutral. As discussed in Gomes et al. (2016); Auclert (2016); Nuno and Thomas (2017), with long-term loans, surprise inflation which would compresses the distribution of real wealth and affect productive efficiency.

$\bar{i}_t^m$ and lets $\bar{i}_t^m$ evolve as follows:

$$\bar{i}_t^m = i_\infty^m + (\bar{i}_0^m - i_\infty^m) \cdot \exp\left(-\bar{\zeta}^{LR} t\right) + (i_{0^-}^m - \bar{i}_0^m) \cdot \exp\left(-\bar{\zeta}^{SR} t\right).$$

This exponential path allows the CB to smoothly deviate from a constant Taylor rule: In the expression, $\bar{\zeta}^{SR}$ governs the speed of adjustment to toward a target $\bar{i}_0^m$ and then $\bar{\zeta}^{LR}$ to its long-run value (see Appendix xxx). Likewise, must modify (15) such that for a period time, the policy rate does not respond to inflation:

$$\eta_t = \eta_\infty + (\bar{\eta}_0 - \eta_\infty) \cdot \exp\left(-\bar{\zeta}^{LR} t\right) + (\eta_{0^-} - \bar{\eta}_0) \cdot \exp\left(-\bar{\zeta}^{SR} t\right).$$

We must limit the response of the Taylor rule to shocks because the promise of lower rates in the future, can lead to an increase in i_t^m as response of the increase in the inflation rate. Thus, we must induce a reduction in η to issolate the transition. Without this modification, the promise of moving $\bar{i}_t^m$ may lead to a short-run increase to inflation that takes i_t^m in the opposite direction—see Werning (2016) and Uribe Schmidt-Grohe (2018).

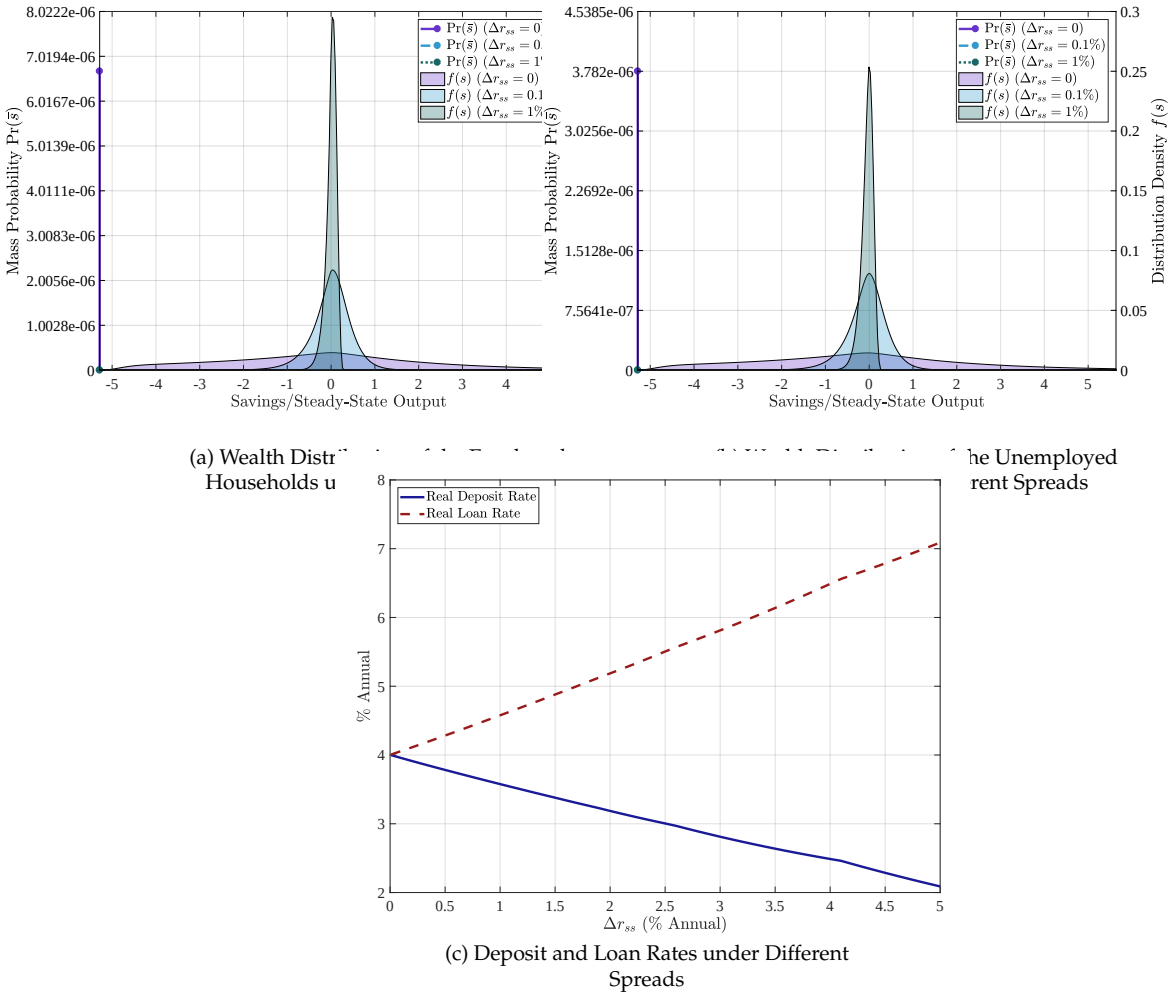
We produce comparisons for the transition paths for different values of $\kappa \in \{0.0001, 0.1, 5\}$. Reductions in the policy rate, lead to reductions in all nominal rates, which produce a decline in real rates together with a corresponding increase in the inflation rate, as in the standard new-Keynesian model. The reduction in real rates, given that production is demand determined, produces an increase in output which is obtained through a decrease in job separations. Interestingly, the volume of credit declines. The reason behind this effect is that the poor can repay their debts faster as they earn more income. The effect goes in the opposite direction of greater consumption smoothing.

4.2 Corridor System and the Credit Channel

Flexible Prices and Steady State. In a flexible price economy, output is determined by the job flows. However, the spread has an effect on insurance, along a transition. Here, we consider the effects of a spread target, Δr on the steady state. The effects apply to the steady state of the economy with price rigidity. Figure 3 reports the real wealth distribution (panel a) and output and real interests (panel b) for different values of Δr. Wider spreads compress the distribution of wealth. The reason for this effect is that the spread acts like a a tax on intermediation. Like any tax, the spread may have a different incidence on borrowers and lenders, depending on the interest-rate elasticity of their savings. A wider spread reduces the real deposit rate and raises the real loans rates. As a result, the spread makes saving less attractive to both borrowers and savers. This is an indication that wider spreads produce worse risk sharing.

Note that although there is monetary neutrality and super-neutrality in the model, monetary policy can affect long-term interest rates. Namely, by supplying scant reserves, the CB can increase

the spread by adding more frictions to financial intermediation. This effect induces worse micro-economic insurance.



(a) Wealth Distr… Households u…

(c) Deposit and Loan Rates under Different Spreads

Figure 3: Steady State Effects of Real Spreads.

Note: This figure depicts the real wealth distributions, real deposit and loan rates for different values of spreads. In panel (a) and (b), the measure of households with assets $\bar{s}$ is in mass probability (left scale), and the measure of households with $s > \bar{s}$ is in probability density (right scale). In panel (c), deposit and loan rates are expressed in annual percentage terms. For all panels, the real spread is expressed in annual percentage terms.

Transitions after changes in Δr. Next, we consider a transition after a reduction Δr_t. Again, the policy is announced at time zero and lasts for a year. The policy eliminates the spread and then lets the spread converge to steady states smoothy. As explained earlier, the policy has a direct effect through the reduction in spreads and an indirect, fiscal effect by increasing transfers. However, the Ricardian effect is small. Because in the New-Keynesian model, the Taylor rule tends to undo the effects of other shocks, we keep i_t^m constant during the length of the exercise, by feeding in a shock to η.

Once the reduction in spreads takes place, the easing of credit spreads reverses the effects. The reduction in the spread has an incidence on both, the real loan and deposit rate. In a flexible price economy, both rates would sandwich together at some point in the middle of their initial values.

Here, the nominal rate is fixed, so the inflationary effect reduces the real deposit rate. Lower real loans rate allows borrowers to abandon their debt limit faster and therefore stimulates aggregate demand. Likewise, for savers, the reduction in rates stimulates their consumption. The expectation of higher future borrowing rates is an offsetting force because there is a desire to reduce debt. The overall effect is an output expansion that is produced by a decline in job separations.

4.3 Negative IOR and the activation of the DZLB

In this section we present the responses for various levels of reductions in IOR, including the one where IOR goes beyond DZLB. The objective is to show that reduction i_t^m beyond the point DZLB is induces a contraction, even though the it is possible to reach negative interest on reserves. When this is the case, the reduction in i_t^m has the effect of a reduction in i_t^m, as in the standard Taylor rule. At the same time, it has the effects of increasing spreads.

Figure 6 reports the dynamic responses in the model corresponding to three values for the reduction the policy rate, the rate $\bar{i}_0^m$. One value leaves the interests in positive territory, the second the interest rate exactly to the point where the DZLB is activated, but not beyond that point. The final response, takes the value beyond the DZLB. The main insight is that the reduction in the policy rate, seizes to be expansionary beyond the point that triggers the DZLB.

4.4 Credit Crunch

Borrowing and Debt Limits. We now study the benefits of relaxing spreads during a credit crunch. To introduce a credit crunch, we modify the model. In addition to the *debt limit* $\bar{s}$, we introduce a potentially time-varying *borrowing limit*, $\tilde{s}_t$. The borrowing limit is triggered before the household reaches its debt limit, $\bar{s} \leq \tilde{s}_t \leq 0$. The idea is that if households reach their borrowing limit, they cannot take on more debt principal, but they can roll it over. That is, in $s \in [\bar{s}, \tilde{s}_t]$, households can refinance their interest payments, but not take more debt. Formally, this means that $ds_t \geq r_t s_t dt$ in $s \in [\bar{s}, \tilde{s}_t]$. Thus, the earlier constraint now reads $c_t dt \leq r_t s_t dt + dw_t$ in $s \in [\bar{s}, \tilde{s}_t]$ and thus, the safe endowment $u_t = L$ is forced in $s \in [\bar{s}, \tilde{s}_t]$. The household's Hamilton-Jacobi-Bellman (HJB) equation is modified to take into account these new constraints.

Intuitively, $\tilde{s}_t$ triggers the inefficient choice earlier. We interpret an increase in $\tilde{s}_t$ as a credit crunch. This distinction between borrowing and debt limits has technical and economic motivations. The technical motivation is that it allows us to study an unexpected credit crunch—an unexpected jump

in the debt limit is now well-defined mathematically.[21] The economic motivation is that if banks wants to cut back on credit, it is convenient to tighten the borrowing limit, but not necessarily the debt limit.[22]

A Credit Crunch. Lets first discuss the transitions produced by a credit crunch, for now holding a fixed spread. We introduce a temporal expected increase in $\tilde{s}_t$, starting from $\tilde{s}_{ss} = \bar{s}$. The borrowing limit is known to tighten to $\tilde{s}_t = 0.8 \cdot \bar{s}$ in a year, and the effect will lasts two years. Figure **??** shows the dynamics after the crunch. The anticipation of the crunch leads to a reduction in credit because it is known that being in debt in the future will be painful. Naturally, borrowers want to pay off their debts, but then savers must hold less deposits. Panel (b) shows how both real deposits and loans fall during the transition. As a result, real deposit rates must fall to discourage savers from savings. The borrowing rate also falls, because the spread is constant, and borrowers are less interest rate sensitive—Panel (c). In the ex ante phase, output actually expands as the mass of agents in the debt limit falls. Once the crunch takes place, a large mass of agents is suddenly in the borrowing-constrained region, $s_t \in [\bar{s}, \tilde{s}_t]$. This forces households in that region to the inefficient choice. The consequence is an immediate output collapse. Output falls continuously as more households are dragged into the borrowing constrained region. The expectation of a recovery produces an increasing path of real interest rates—because borrowing-constrained households roll over a greater stock of debt. Credit continues to decrease until it reverses as the end of the crunch approaches.

The following figures plot the transition paths of a credit crunch shock.

5 Normative Analysis: Optimal use of the Credit Channel

Spread Management during a Credit Crunch We simulate the model to investigate the impacts of four policy choices during credit crunch: (1) Close spread to 0; (2) Reduce IOER to DZLB such that the nominal deposit rate is zero during credit crunch, and the credit spread is constant over time; (3) Reduce IOER to DZLB and close spread to 0 such that the IOER i_t^m, the nominal deposit rate i_t^a

[21] Suppose we want to study a credit crunch by an unexpected tightening of the debt limit. If there is an unexpected change in the debt limit, there would be a positive mass of households violating their debt limits because income flows continuously. This inconvenience does not apply when the borrowing limit $\tilde{s}_t$ moves unexpectedly. In the latter case, households now face a problem insuring risk, but are not forced to reduce their debt stock immediately. This is a technical assumption to circumvent an issue faced by models with debt limits. For example, Guerrieri and Lorenzoni (2017) must study a gradual shock to debt limits precisely to leave agents enough time to abandon their borrowing limits.

[22] When a bank extends a loan principal, it increases its liabilities. This is not true about a rollover. In the case of a rollover, banks earn interest that increases equity, but not liabilities. During financial crises, banks will want to roll over debt, although they are unwilling to extend principal because the latter consumes regulatory capital. In addition, if loan repayment is suddenly forced, it can trigger default. Defaults are costly for banks, because they lead to underwritings that also subtract regulatory capital. The formulation here is motivated by these observations, although their explicit modeling is outside the scope of the paper. This phenomenon is called evergreening. We do not model this explicitly, but we are guided by this economic interpretation. Our constraint is consistent with the interpretation. Caballero et al. (2008) present a model of evergreening.

and the spread Δr_t are all equal to 0 during credit crunch; (4) Reduce IOER below DZLB such that we reduce IOER to 0.5% below the level in policy choice (2), so there is an increase in spread during credit crunch.

In all cases below, we consider four scenarios: the economy starts with a steady state with the same IOER, $i^m_{ss} = 1\%$, and a different value of spread $\Delta r_{ss} \in \{0.5\%, 0.75\%, 1\%, 1.25\%\}$. Due to the superneutrality of money, all the aggregate real variables except the real credit (borrowing and saving) have same values across the four scenarios. This provides us a consistent benchmark for comparing the impacts of different policy choices. In all simulations, the policy choices mentioned above only take place during credit crunch, and return back to steady-state values outside the credit crunch.

The main conclusion of the simulations: (1) Starting with a higher steady-state spread is always good to offset the decline of output during credit crunch, no matter which policy we use. (2) Going below DZLB does no good to the real economy, since it simply raises the spread that contracts credit. (3) The ex ante welfare of the transition path is non-monotonic in the spreads. We measure the welfare loss as the percentage deviation of the time-0 aggregate certainty equivalence from the steady-state total output. Among all the policy scenarios, the welfare loss is minimized at $\Delta r_{ss} = 0.75\%$. This implies a welfare trade-off of opening spread ex ante: with a higher steady-state spread, the steady-state welfare loss is larger, while the welfare loss during credit crunch is smaller due to the power of spread to offset the shock of credit crunch.

The following table compares the transition path welfare loss across the policy responses:

Table 10: Welfare Loss of Policies During Credit Crunch (% deviation of CE from Y_{ss})

Steady-State Spread Δr_{ss}	Transition Path Welfare Loss			
	0.5%	0.75%	1%	1.25%
Close Spread	0.0872	0.0841	0.0851	0.0865
Reduce IOER to DZLB	0.0806	0.0780	0.0795	0.0811
Reduce IOER Below DZLB	0.0808	0.0784	0.0800	0.0811
Reduce IOER to DZLB and Close Spread	0.0793	0.0760	0.0769	0.0783
Steady-State Welfare Loss	0.0245	0.0292	0.0335	0.0376

Discount Factor Shocks. Appendix xxx studies the effects of a discount factor shock.

Ancticipated Shocks: The Risky Steady State. We can conduction the same exercises, allowing the shock to be anticipated. The following table reports the welfare loss (in terms of certainty equivalence) at time 0 and the following figure plots the transition paths of all scenarios.

Table 5: Welfare Loss of Closing Spread During Credit Crunch in Risky Steady State Scenario

Scenario of Δr	0.5%	0.75%	1%	1.25%
Transition Path Welfare Loss (% deviation of CE from Y_{ss})	0.0481	0.0479	0.0497	0.0512
Steady State Welfare Loss (% deviation of CE from Y_{ss})	0.0227	0.0271	0.0311	0.0349

6 Conclusion

In the final paragraph of the introduction to his collected works on monetary economics, Lucas (2013), Robert E. Lucas writes: "Now, toward the end of my career as at the beginning, I see myself as a monetarist. My contributions to monetary theory have been to incorporate the quantity theory into modern modeling. For the empirically well established predictions —long-run links— this job has been accomplished. On the harder questions of monetary economics — the real effects of monetary instability, the roles of inside and outside money, this work contributes examples but little in empirically successful models. It is understandable that in the leading operational macroeconomic models today— the RBC and the New Keynesian models—money as a measurable magnitude plays no role at all, but I hope we can do better than this in the future."

This paper is one of the many attempts to let money play the role that Lucas refers to. The model here is actually a descendant of one of Lucas's early monetary models, Lucas (1980). Here, outside money (reserves) is an input for inside money creation (deposits and loans). The current attempt tries to be explicit about the implementation of MP. The novelty is that MP operates by controlling spreads. If we are open to accepting that idea, we may challenge some traditional views. For example, we may challenge the idea that MP is long-run neutral[23] and that inflation and monetary aggregates are tied together, which represents two working restrictions in conventional empirical work. The model rationalizes several empirical regularities. For example, the model rationalizes the presence of a liquidity effect and a higher loan than deposit rate elasticity to policy changes. A normative message is that managing spreads is desirable: although spreads limit risk sharing, they may improve efficiency. In the case of a credit crunch, countercyclical spreads implemented via open market operations are a desirable policy that does not compromise inflation. Hence, the advice to remain with active corridor systems.

[23]This feature is also true in other incomplete market economies with money; the reason is not the spread, but the effect of real monetary balances on credit markets.

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Figures for Positive Analysis

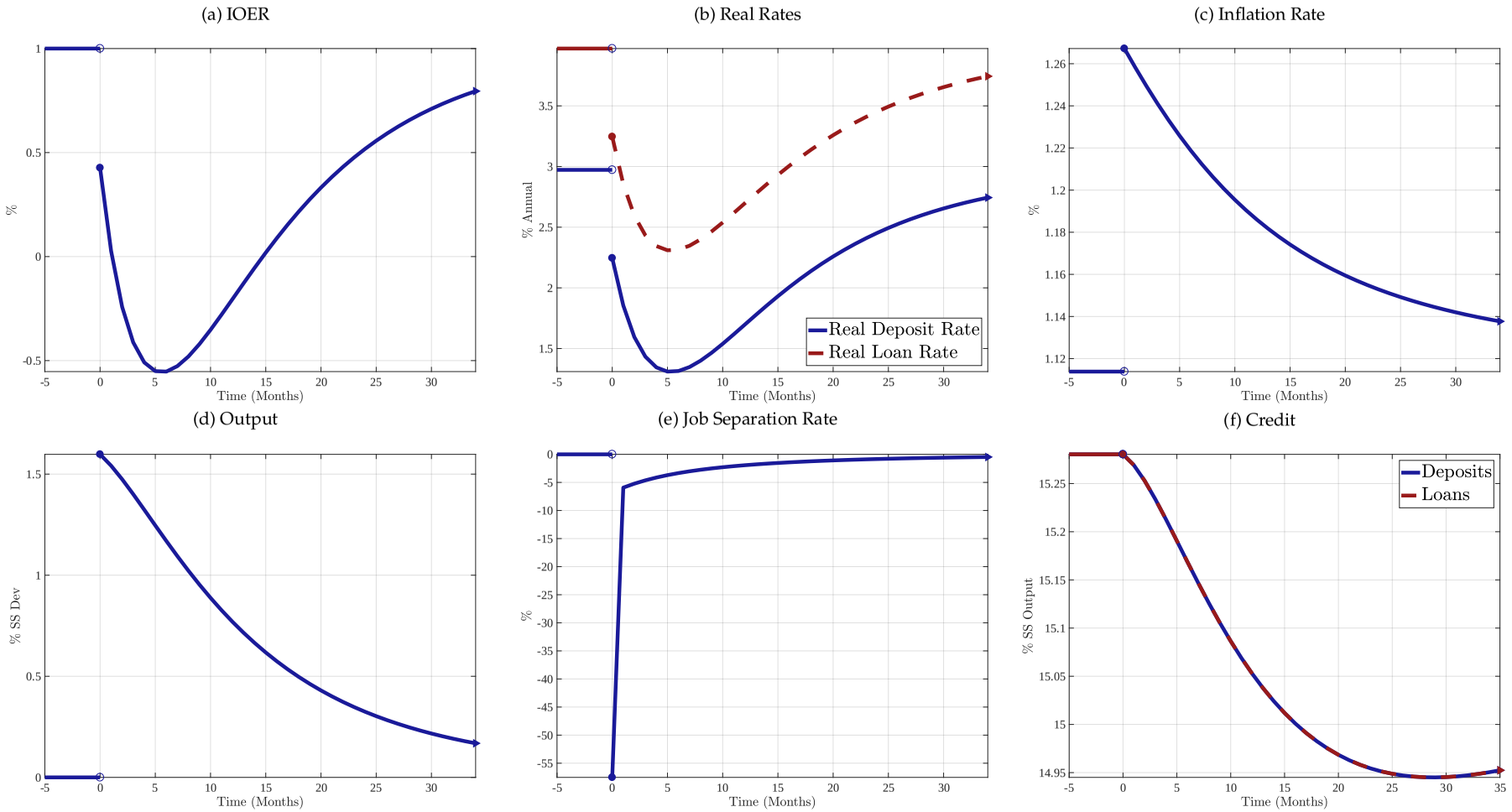


Figure 4: Transition Dynamics after a IOR Reduction (Interest Rate Target).

Note: The figure reports the responses to rates, inflation, output, job separation rate and credit after an unanticipated IOR reduction. In panels (a), (b), (c) and (e), all the rates are expressed in annual percentages. In panels (d), the aggregate output is expressed in percentage deviations from the steady state. In panel (f), the aggregate credit is expressed in credit-to-steady-state output ratio. The reduction in the IOR is unanticipated at time zero. Starting time 0, the path of IOR follows the modified Taylor Rule, where the short-run path of IOR is independent of the inflation rate . The IOR is initially fixed at 1% annually before time 0.

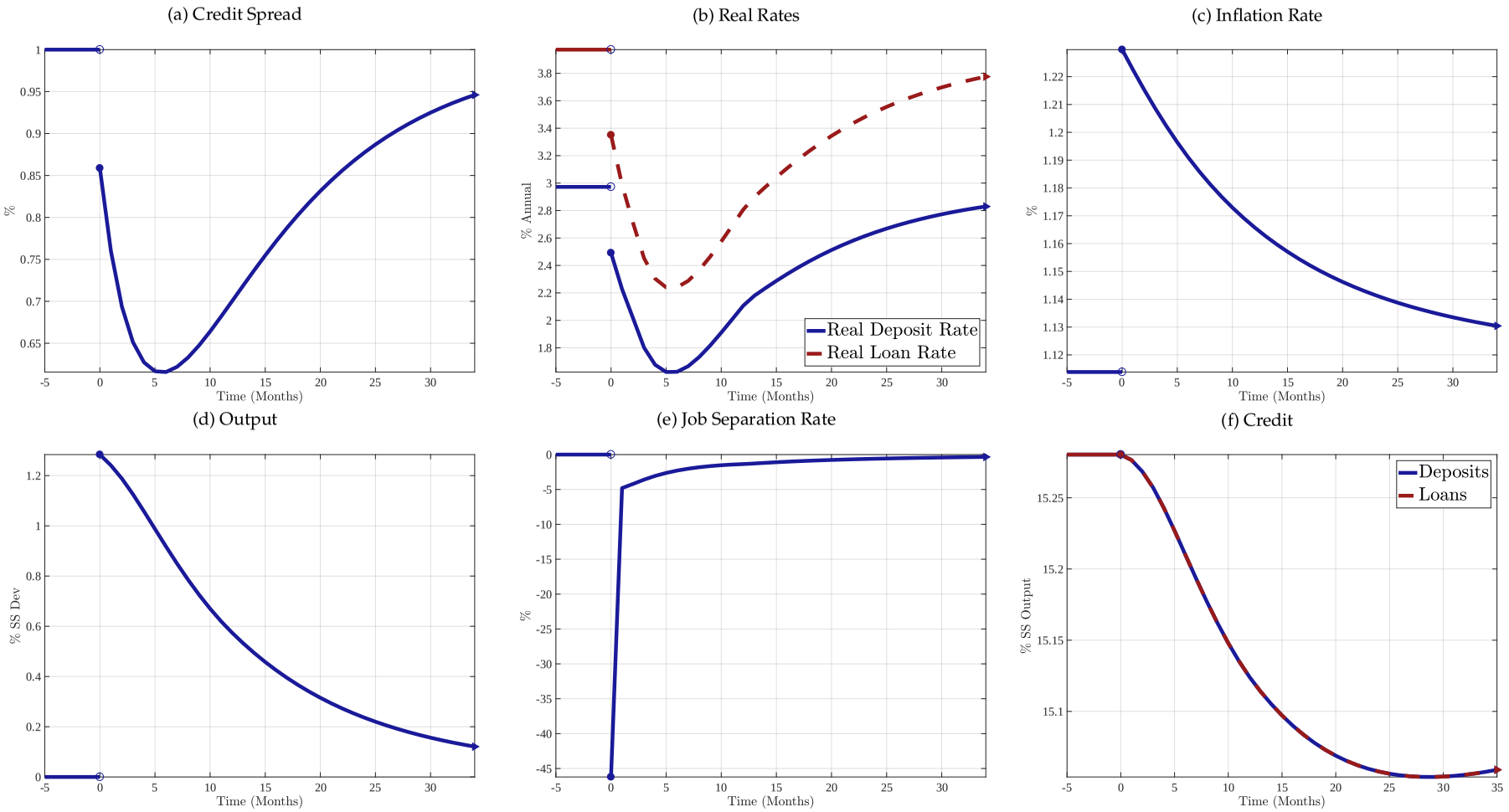


Figure 5: Transition Dynamics after a Credit Spread Reduction (Credit Target).

Note: The figure reports the responses to rates, inflation, output, job separation rate and credit after an unanticipated credit spread reduction. In panels (a), (b), (c) and (e), all the rates are expressed in annual percentages. In panels (d), the aggregate output is expressed in percentage deviations from the steady state. In panel (f), the aggregate credit is expressed in credit-to-steady state output ratio. The reduction in the credit spread is unanticipated at time zero. Starting time 0, the path of IOR follows the modified Taylor Rule, where the short-run path of IOR is independent of the inflation rate . The IOR is initially fixed at 1% annually before time 0.

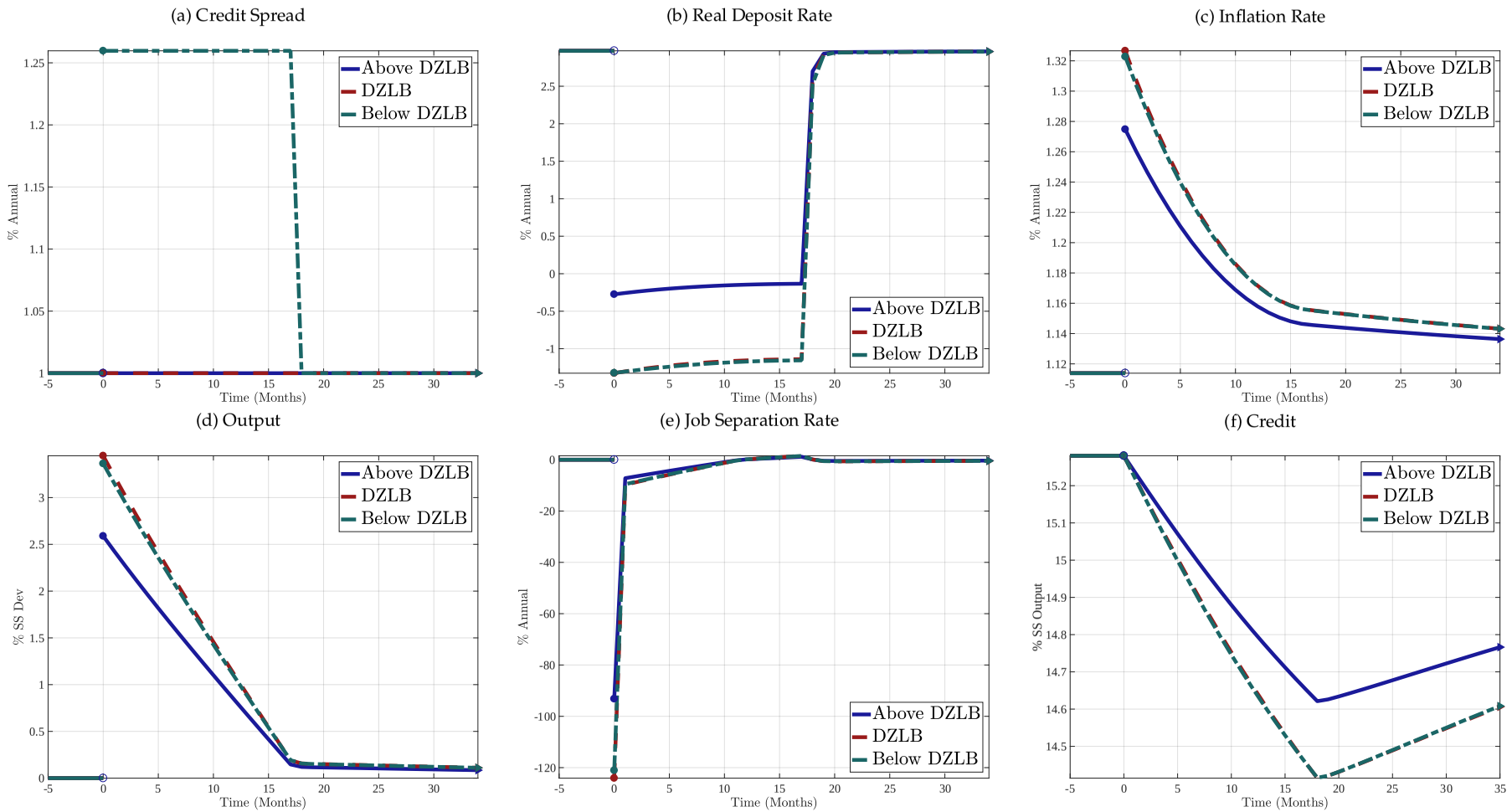


Figure 6: Transition Dynamics after a IOR Reduction (Negative IOR and DZLB).

Note: The figure reports the responses of rates, inflation, output, job separation rate and credit after an unanticipated IOR reduction. In panels (a), (b), (c) and (e), all the rates are expressed in annual percentages. In panel (d), the aggregate output is expressed in percentage deviations from the steady state. In panel (f), the aggregate credit is expressed in credit-to-total output ratio. The reduction in the IOR is unanticipated at time zero and lasts for 18 months. Starting from time 0, the path of IOR follows the modified Taylor Rule. The IOR is initially fixed at 1% annually before time 0. Scenario "DZLB" means the IOR is reduced to the level that reduces the nominal deposit rate to 0. Scenario "Above DZLB" means the IOR is reduced to the level higher than the DZLB level. Scenario "Beyond DZLB" means the IOR is reduced to the level lower than the DZLB level.

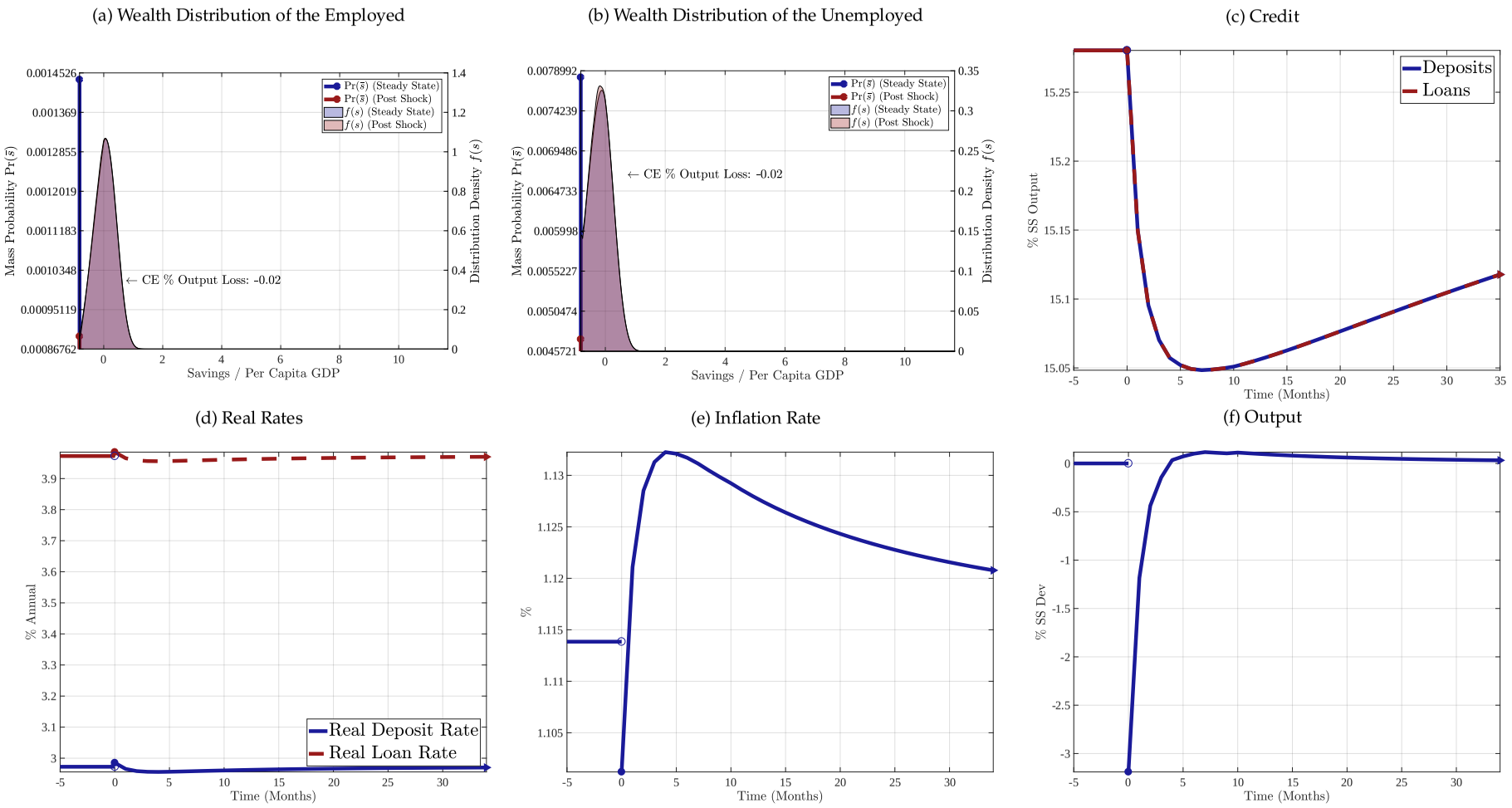
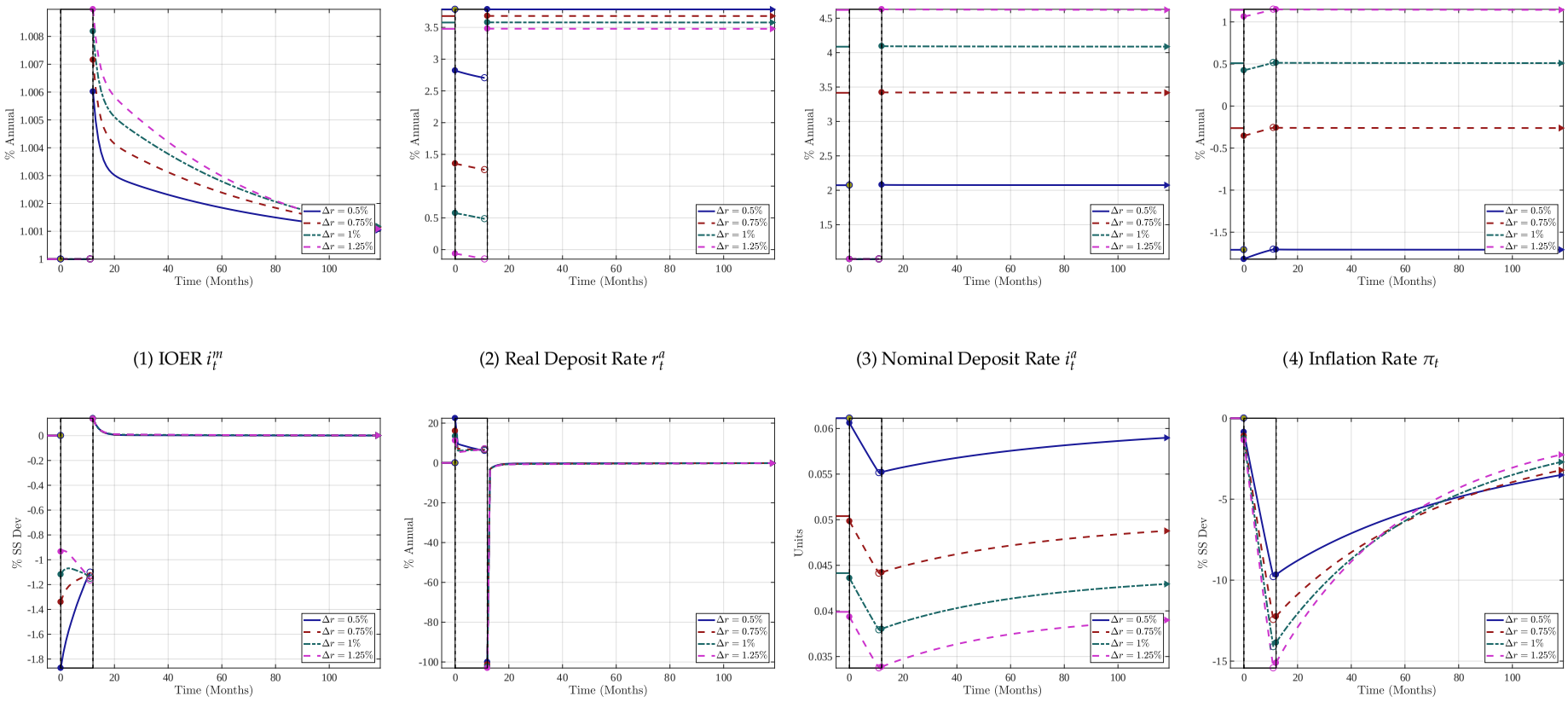


Figure 7: Transition Dynamics after a Credit Crunch.

Note: The figure reports the real wealth distribution, and the responses to credit, rates, inflation and output after an unanticipated credit crunch. In panels (a) and (b), the measure of households with assets $\bar{s}$ is in mass probability (left scale), and the measure of households with $s > \bar{s}$ is in probability density (right scale). The Certainty Equivalent (CE) % loss is expressed in the percentage deviation of aggregate certainty equivalent after the announcement. In panels (c) and (f), the aggregate deposits, loans and output are expressed in percentage deviations from the steady state. In panels (d) and (e), the real rates and inflation rate are expressed in annual percentages. The credit crunch is unanticipated at time zero (at the first vertical dashed line), and lasts for one year (between the two vertical dashed lines). During the credit crunch, the borrowing constraint $\bar{s}_t = 0.05\bar{s}$. The net income from credit spread is returned back to households as lump-sum transfer.

Figures for Normative Analysis

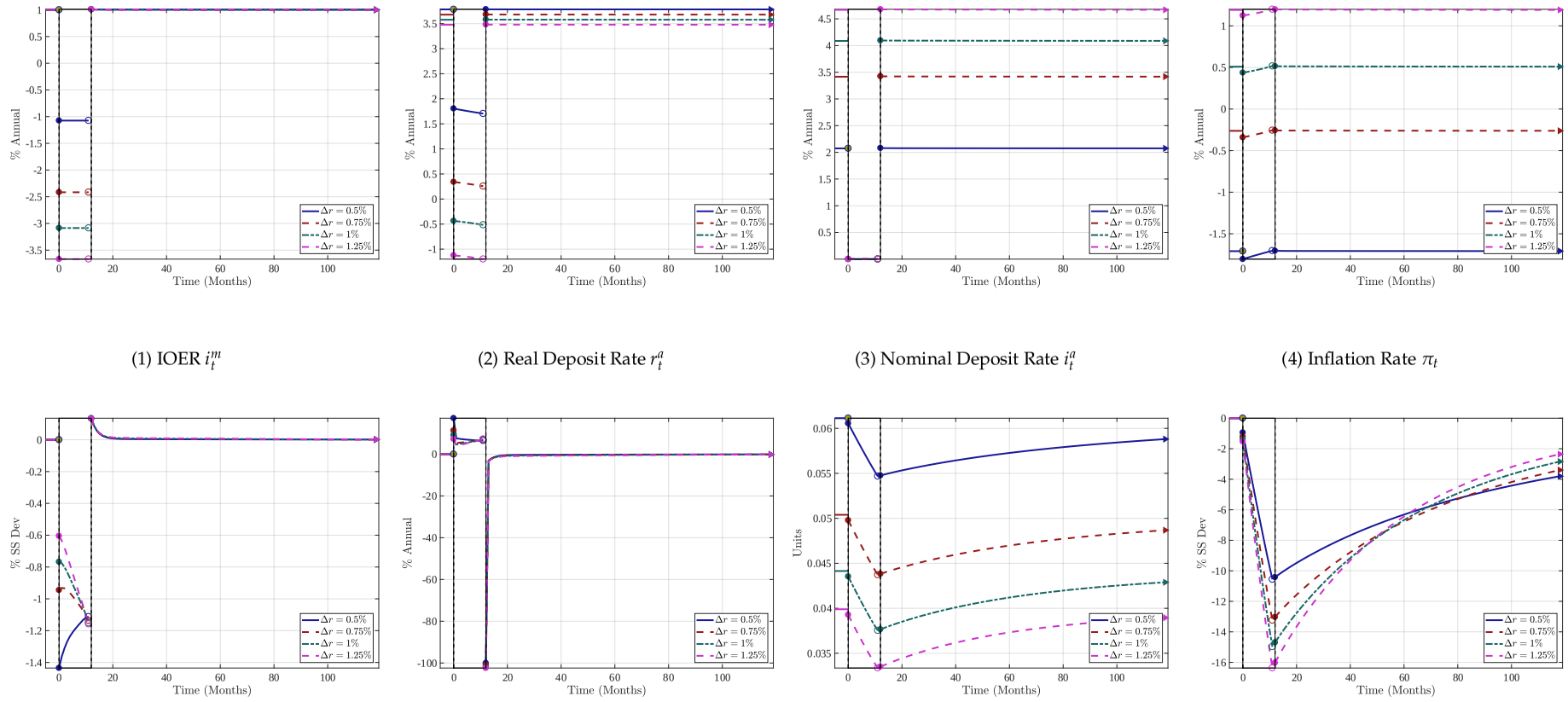


(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 8: Transition Paths of Credit Crunch and Closing Spread

Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated credit crunch and credit spread reduction. The credit spread is reduced to zero during credit crunch, and increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\tilde{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set $i_{ss}^m = 1\%$ for all scenarios. For the monetary policy during credit crunch, we set $\Delta r_t = 0$ and do not change i_t^m. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

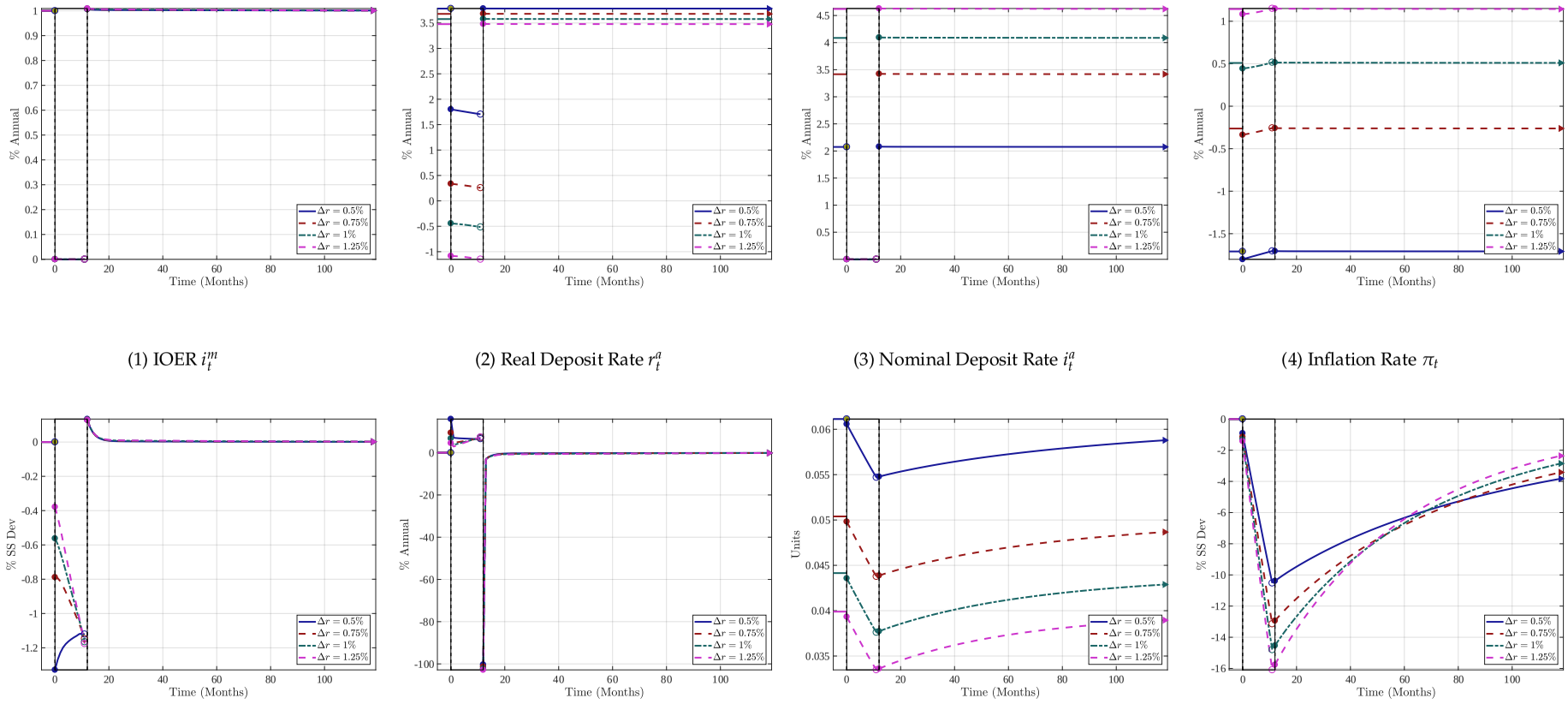


(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 9: Transition Paths of Credit Crunch and Reducing IOER to DZLB

Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated credit crunch and IOER reduction. During the credit crun we set $\bar{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\,\chi_{ss}^-\right]$ and do not change Δr_t. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\tilde{s}' = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

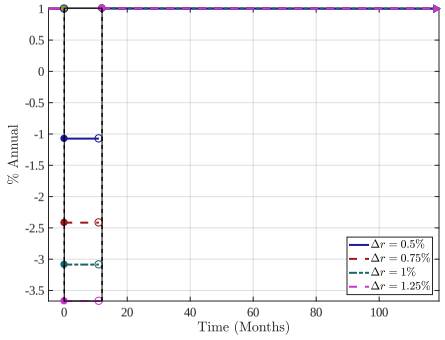


(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

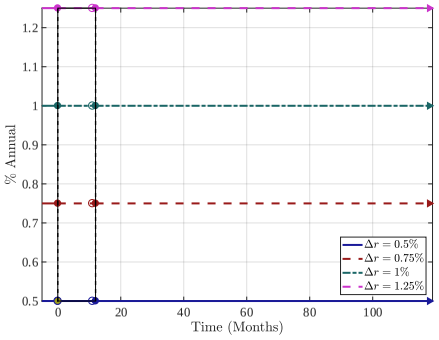
(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 10: Transition Paths of Credit Crunch, Reducing IOER to DZLB and Closing Spread

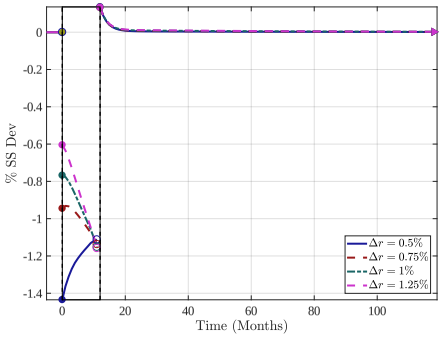
Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated credit crunch with a simultaneous reduction in credit spread and IOER. During the credit crun we set $\Delta r_t = \bar{i}_0^m = 0$. Out of the credit crunch we set $\Delta r_t \equiv \Delta r_{ss}$ for all scenariosand increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\bar{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.



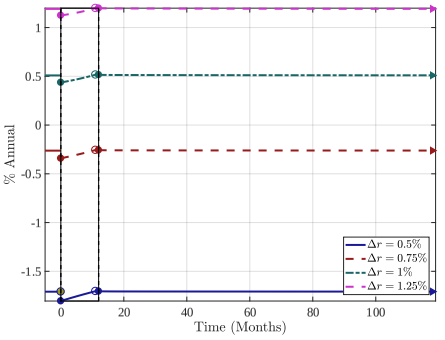
(1.a) IOER i_t^m (Reducing IOER to DZLB)



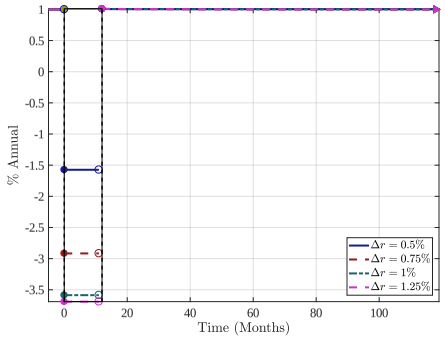
(2.a) Spread Δr_t (Reducing IOER to DZLB)



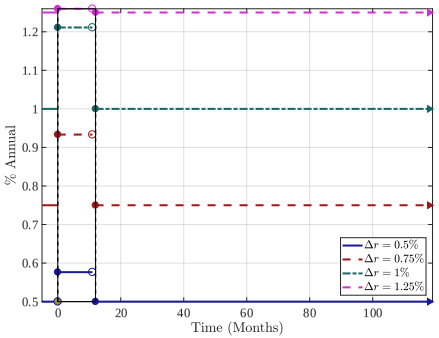
(3.a) Output (% Deviation from Steady State, Reducing IOER to DZLB)



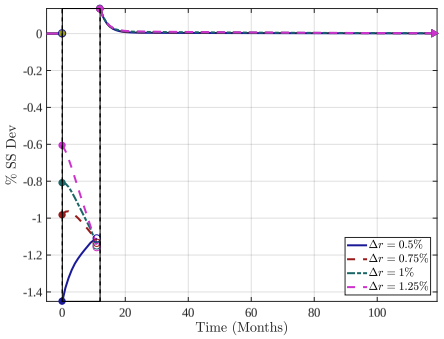
(4.a) Inflation Rate π_t (Reducing IOER to DZLB)



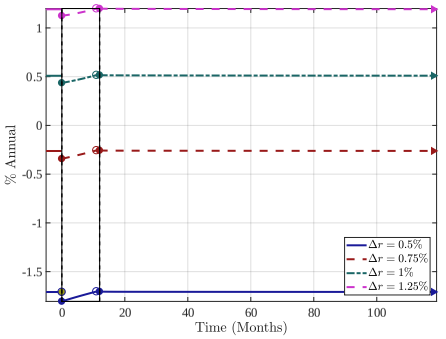
(1.b) IOER i_t^m (Reducing IOER below DZLB)



(2.b) Spread Δr_t (Reducing IOER below DZLB)



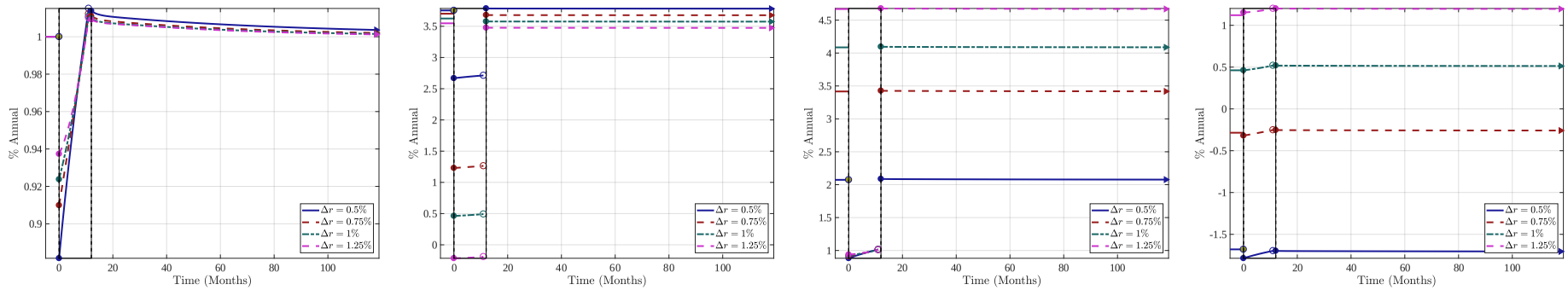
(3.b) Output (% Deviation from Steady State, Reducing IOER below DZLB)



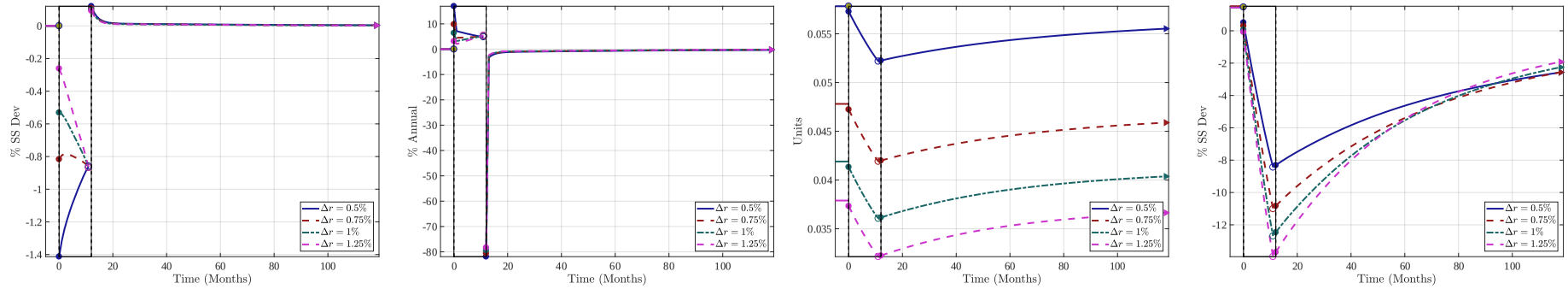
(4.b) Inflation Rate π_t (Reducing IOER below DZLB)

Figure 11: Transition Paths of Reducing IOER to DZLB vs Reducing IOER below DZLB

Note: The figure reports the paths of IOER, credit spread, aggregate output and inflation rate after an unanticipated credit crunch and IOER reduction. In the panels on the left column, we set $\tilde{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\chi_{ss}^-\right]$ and do not change Δr_t during credit crunch. In the panels on the right column, we set $\tilde{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\chi_{ss}^-\right]$-$\Delta i_0$ and changes Δr_t such that i_t^a does not fall below zero during credit crunch. The value of $\Delta i_0 = \min\left\{0.5\%, \frac{1}{2}\left[\chi^+(+\infty) - \chi_{ss}^+ + (1-\delta)\left(\chi^-(+\infty) - \chi_{ss}^-\right)\right]\right\}$. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\tilde{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (3.a) and (3.b), the aggregate output is expressed in percentage deviations from the steady state.



(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t



(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 12: Transition Paths of Credit Crunch and Closing Spread in Risky Steady State

Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated credit crunch and credit spread reduction. The credit crunch arrives as a Poisson shock for only once. The credit spread is reduced to zero during credit crunch, and increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\bar{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set $i_{ss}^m = 1\%$ for all scenarios. For the monetary policy during credit crunch, we set $\Delta r_t = 0$ and do not change i_t^m. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

Online Appendix

A Accounting in the Model

A.1 Balance Sheets

Household Balance Sheet. The household's balance sheet in in nominal terms is structured as:

Assets	Liabilities
m_t^h	l_t^h
a_t^h	$P_t s_t$

Bank Balance Sheet. The balance sheet of an individual bank is structured as:

Assets	Liabilities
m_t^b	a_t^b
l_t^b	

CB Balance Sheet. The balance sheet of the CB is structured as:

Assets	Liabilities
L_t^f	M_t
	E_t

Accounting of OMO. To interpret OMO as purchases of government debt, consider F_t as an outstanding amount of nominal bonds issued by a fiscal authority. Let $F_t^{cb} < F_t$ be the stock of bonds held at the CB. In that case, the balance sheet of the consolidated government is

Assets	Liabilities
F_t^{cb}	$M_t + F_t$
	E_t

$=$

Assets	Liabilities
$F_t^{cb} - F_t$	M_t
	E_t

Thus, $L_t^f = F_t^{cb} - F_t < 0$ is the stock of government bonds held by banks and E_t is the stock of government liabilities net of CB purchases. A conventional open-market operation is simply an increase in F_t^{cb} funded with an increase in M_t. From the government's income flow, we can see that this operation would yield profits to the CB if there's a spread $i_t^l > i_t^m$. Figures A.1 and A.1 present the consolidated balance sheets.

Monetary Aggregates. The monetary aggregates are given by, M_t, the monetary base, $M0_t$, the currency and $M1_t \equiv A_t^b + M0_t$, the highest monetary aggregate.

Timeline of Interbank transactions. Figure A.1 presents the accounting for banks, within a Δ time interval. Unlucky banks get hit by negative withdrawal shocks, which can lead them to a negative balance of reserves in the period. That bank mus cover the position by the end of the interval by borrowing funds from other banks, or from the discount window.

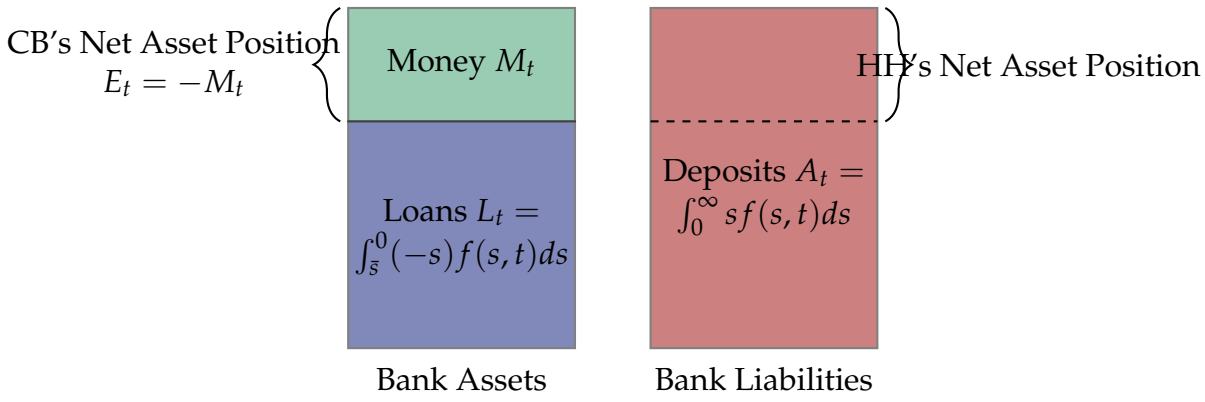


Figure 13: Baseline Bank Balance Sheet

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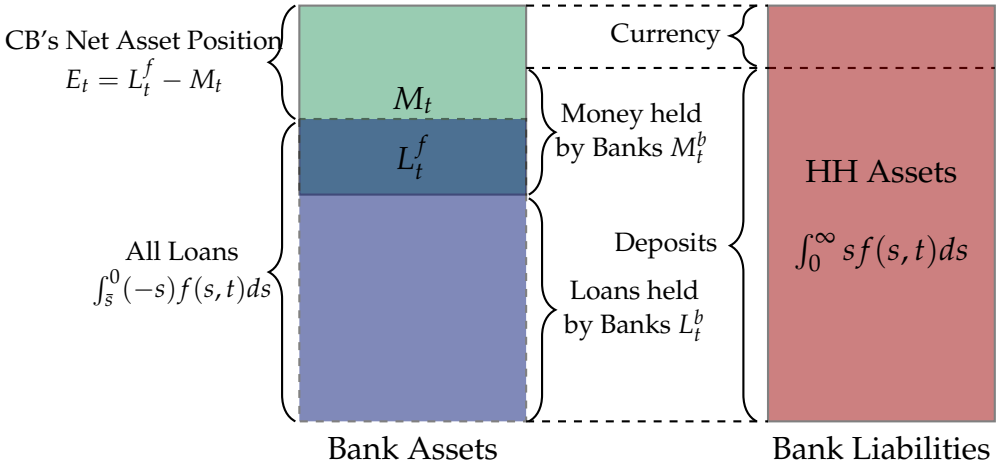
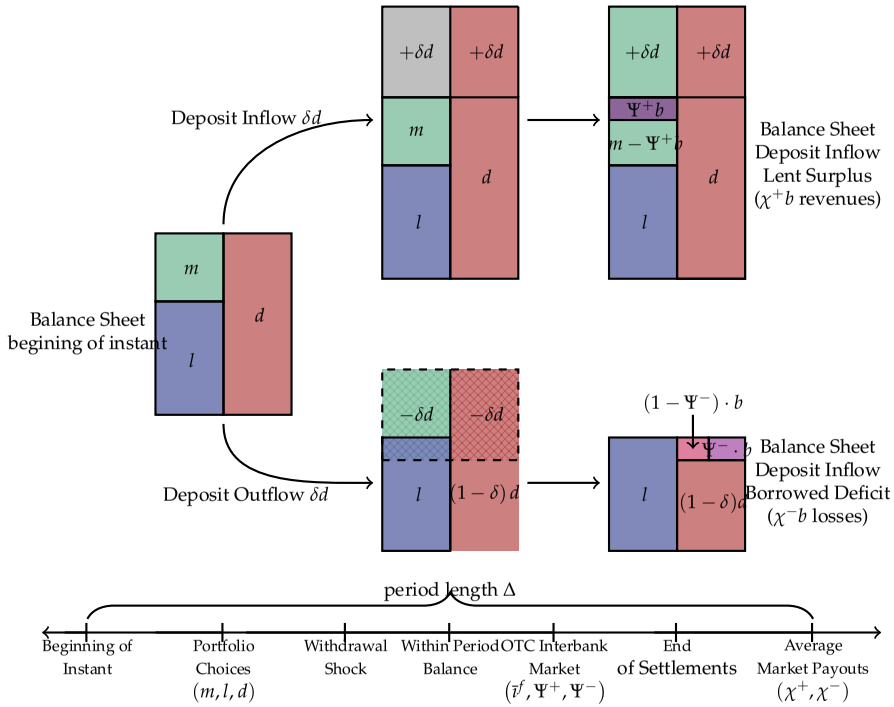


Figure 14: Bank Balance Sheet under ZLB/OMO

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Deposit Inflow δd
Balance Sheet
begining of instant
m
l
d
$+\delta d$
$+\delta d$
m
l
d
$\Psi^+ b$
$m - \Psi^+ b$
Balance Sheet
Deposit Inflow
Lent Surplus
($\chi^+ b$ revenues)
Deposit Outflow δd
$-\delta d$
$-\delta d$
l
$(1-\delta)\, d$
$(1-\Psi^-)\cdot b$
$\Psi^- \cdot b$
l
$(1-\delta)d$
Balance Sheet
Deposit Inflow
Borrowed Deficit
($\chi^- b$ losses)
period length Δ
Beginning of Instant
Portfolio Choices (m, l, d)
Withdrawal Shock
Within Period Balance
OTC Interbank Market $(\bar{r}^f, \Psi^+, \Psi^-)$
End of Settlements
Average Market Payouts (χ^+, χ^-)

A.2 Flow of Funds Identities

Lemma 1 *If the deposit, loans and money markets clear, then:*

$$P_t \int_0^{\infty} sf(s,t)ds = -P_t \int_{\bar{s}}^{0} sf(s,t)ds - E_t. \tag{23}$$

Proof. The deposits and loans markets clearing condition requires:

$$A_t^b = \int_0^{\infty} a_t^h(s) f(s,t) ds \tag{24}$$

$$L_t^b + L_t^f = \int_{\bar{s}}^{0} l_t^h(s) f(s,t) ds, \tag{25}$$

and clearing in the money market requires:

$$M_t^b + M0_t = M_t \tag{26}$$

We also have that the budget constraint (balance sheet) of banks satisfies the following identity:

$$A_t^b = L_t^b + M_t^b. \tag{27}$$

Real household assets are held as nominal deposits or currency, hence:

$$P_t \int_0^{\infty} sf(s,t)ds = \int_0^{\infty} a_t^h(s) f(s,t) ds + M0_t. \tag{28}$$

and, similarly for liabilities:

$$-P_t \int_{\bar{s}}^{0} sf(s,t)ds = \int_{\bar{s}}^{0} l_t^h(s) f(s,t) ds. \tag{29}$$

Once we combine (24), (25) and (27), we obtain a single condition:

$$\int_0^{\infty} a_t^h(s) f(s,t) ds = \int_{\bar{s}}^{0} l_t^h(s) f(s,t) ds - L_t^f + M_t^b. \tag{30}$$

This condition can be expressed in terms of real household wealth, with the use of (28) and (29):

$$P_t \int_0^{\infty} sf(s,t)ds = -P_t \int_{\bar{s}}^{0} sf(s,t)ds - L_t^f + M_t^b + M0_t.$$

If we use the money market clearing-condition, (16), and employ the definition of net-asset position of the CB, we obtain (23). **QED.**

B Interbank-Market Equilibrium and Implementation Figures

According to Bianchi and Bigio (2017b), the trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \theta\left(1-e^{-\lambda}\right), \qquad \psi^{-}(\theta) \equiv 1-e^{-\lambda}.$$

The resulting average interbank market rate is determined by the average of Nash bargaining over the positions and is given by:

$$\bar{i}^{f}(\theta, i^{m}, \iota) \equiv i^{m}+\iota-\left(1-\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}\right)\left(\frac{\theta/\bar{\theta}(\theta)}{1-\theta}\right)\left(\frac{\iota}{e^{\lambda}-1}\right) \tag{31}$$

and the average liquidity-yield functions are

$$\chi^{+}(\theta, \iota)=\iota\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}(\theta)^{1-\eta}-\theta}{\bar{\theta}(\theta)-1}\right) \text{ and } \chi^{-}(\theta, \iota)=\iota\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}(\theta)^{1-\eta}-1}{\bar{\theta}(\theta)-1}\right), \tag{32}$$

where η is a parameter associated with the bargaining power of banks with reserve deficits, and $\bar{\theta}(\theta)$ is the end-of-day market tightness:

$$\bar{\theta}(\theta)=\left(1+\left(\theta^{-1}-1\right)\exp(\lambda)\right)^{-1}. \tag{33}$$

Thus, the path for $\left\{\psi_t^{+}, \psi_t^{-}, \bar{i}_t^{f}, \chi_t^{+}, \chi_t^{-}\right\}$ is given by $\psi_t^{+} \equiv \psi^{+}(\theta_t)$, $\psi_t^{-} \equiv \psi^{-}(\theta_t)$, $\bar{i}_t^{f} \equiv \bar{i}^{f}(\theta_t, i_t^{m}, \iota_t)$, $\chi_t^{+} \equiv \chi^{+}(\theta_t, \iota_t)$ and $\chi_t^{-} \equiv \chi^{-}(\theta_t, \iota_t)$. In the paper, we set $\eta = 1/2$. By replacing $\bar{\theta}(\theta)$ with (33) and setting $\theta < 1$, equations (31) and (32) reduce to (2) and (3).

B.1 Simplified Version

According to Bianchi and Bigio (2017b), the trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \theta(1-\exp(\lambda)), \qquad \psi^{-}(\theta) \equiv 1-\exp(\lambda).$$

Let

$$\bar{\theta}=\left(1+\left(\theta^{-1}-1\right)\exp(\lambda)\right)^{-1}.$$

Then,

$$\frac{\bar{\theta}}{\theta}=\frac{\theta}{\theta(\theta+(1-\theta)\exp(\lambda))}=\frac{1}{\theta+(1-\theta)\exp(\lambda)}$$

Then,

$$1-\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}=1-\left(\frac{1}{\theta+(1-\theta)\exp(\lambda)}\right)^{\eta}$$

And

$$1-\bar{\theta}=1-\frac{1}{(1+(\theta^{-1}-1)\exp(\lambda))}=\frac{(1-\theta)\exp(\lambda)}{\theta+(1-\theta)\exp(\lambda)}.$$

Two additional terms:

$$\theta^{\eta}\bar{\theta}^{1-\eta}=\frac{\theta^{\eta}\left(1+\left(\theta^{-1}-1\right)\exp(\lambda)\right)^{\eta}}{(1+(\theta^{-1}-1)\exp(\lambda))}=\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(1+(\theta^{-1}-1)\exp(\lambda))}=\theta\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(\theta+(1-\theta)\exp(\lambda))}.$$

Then:

$$h^{-}=1-\theta^{\eta}\bar{\theta}^{1-\eta}=\left(1-\theta\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(\theta+(1-\theta)\exp(\lambda))}\right)$$

and

$$h^{+}=\theta-\theta^{\eta}\bar{\theta}^{1-\eta}=\theta\left(1-\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(\theta+(1-\theta)\exp(\lambda))}\right)$$

As a result:

$$\chi^{+}=\iota\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}h^{+}\left(\frac{1}{1-\bar{\theta}}\right)=\left(\frac{1}{\theta+(1-\theta)\exp(\lambda)}\right)^{\eta}\frac{\theta\left(1-\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(\theta+(1-\theta)\exp(\lambda))}\right)}{\frac{(1-\theta)\exp(\lambda)}{\theta+(1-\theta)\exp(\lambda)}}=$$

and

$$\chi^{-}=\iota\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}h^{-}\left(\frac{1}{1-\bar{\theta}}\right)=\left(\frac{1}{\theta+(1-\theta)\exp(\lambda)}\right)^{\eta}\frac{\left(1-\theta\frac{(\theta+(1-\theta)\exp(\lambda))^{\eta}}{(\theta+(1-\theta)\exp(\lambda))}\right)}{\frac{(1-\theta)\exp(\lambda)}{\theta+(1-\theta)\exp(\lambda)}}$$

Define:

$$\gamma=(1-\theta)\exp(\lambda)$$

Then

$$\chi^{+}=\iota\left(\frac{1}{\theta+\gamma}\right)^{\eta}\frac{\theta\left(1-\frac{(\theta+\gamma)^{\eta}}{\theta+\gamma}\right)}{\frac{\gamma}{\theta+\gamma}}=\iota\frac{\theta}{\gamma}(\theta+\gamma)^{-\eta}\left((\theta+\gamma)-(\theta+\gamma)^{\eta}\right)=\iota\theta\frac{\left((\theta+(1-\theta)\exp(\lambda))^{1-\eta}-1\right)}{(1-\theta)\exp(\lambda)}.$$

and

$$\chi^{-}=\iota\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}h^{-}\left(\frac{1}{1-\bar{\theta}}\right)=\left(\frac{1}{\theta+\gamma}\right)^{\eta}\frac{\left(1-\theta\frac{(\theta+\gamma)^{\eta}}{\theta+\gamma}\right)}{\frac{\gamma}{\theta+\gamma}}=\iota\frac{\left((\theta+(1-\theta)\exp(\lambda))^{1-\eta}-\theta\right)}{(1-\theta)\exp(\lambda)}.$$

Therefore, in summary:

$$\chi^{-} = \iota \frac{(\theta + (1-\theta)\exp(\lambda))^{1-\eta} - \theta}{(1-\theta)\exp(\lambda)}.$$

and

$$\chi^{+} = \iota \frac{\theta(\theta + (1-\theta)\exp(\lambda))^{1-\eta} - \theta}{(1-\theta)\exp(\lambda)}.$$

The resulting average interbank market rate is determined by the average of Nash bargaining over the positions and is given by:

$$\bar{i}^{f}(\theta, i^{m}, \iota) \equiv i^{m} + \iota \frac{(\theta + (1-\theta)\exp(\lambda))^{1-\eta} - 1}{1 - \exp(\lambda)} \tag{34}$$

and the average liquidity-yield functions are

$$\chi^{+}(\theta, \iota) = \iota \left(\frac{\bar{\theta}}{\theta}\right)^{\eta} \left(\frac{\theta - \theta^{\eta}\bar{\theta}^{1-\eta}}{1-\bar{\theta}}\right) \text{ and } \chi^{-}(\theta, \iota) = \iota \left(\frac{\bar{\theta}}{\theta}\right)^{\eta} \left(\frac{1 - \theta^{\eta}\bar{\theta}^{1-\eta}}{1-\bar{\theta}}\right), \tag{35}$$

where η is a parameter associated with the bargaining power of banks with reserve deficits, and $\bar{\theta}(\theta)$ is the end-of-day market tightness:

Thus, the path for $\left\{\psi_t^{+}, \psi_t^{-}, \bar{i}_t^{f}, \chi_t^{+}, \chi_t^{-}\right\}$ is given by $\psi_t^{+} \equiv \psi^{+}(\theta_t)$, $\psi_t^{-} \equiv \psi^{-}(\theta_t)$, $\bar{i}_t^{f} \equiv \bar{i}^{f}(\theta_t, i_t^{m}, \iota_t)$, $\chi_t^{+} \equiv \chi^{+}(\theta_t, \iota_t)$ and $\chi_t^{-} \equiv \chi^{-}(\theta_t, \iota_t)$. In the paper, we set $\eta = 1/2$. By replacing $\bar{\theta}(\theta)$ with (**??**) and setting $\theta < 1$, equations (34) and (35) reduce to (2) and (3).

B.2 Additional Implementation Figures: CB Income

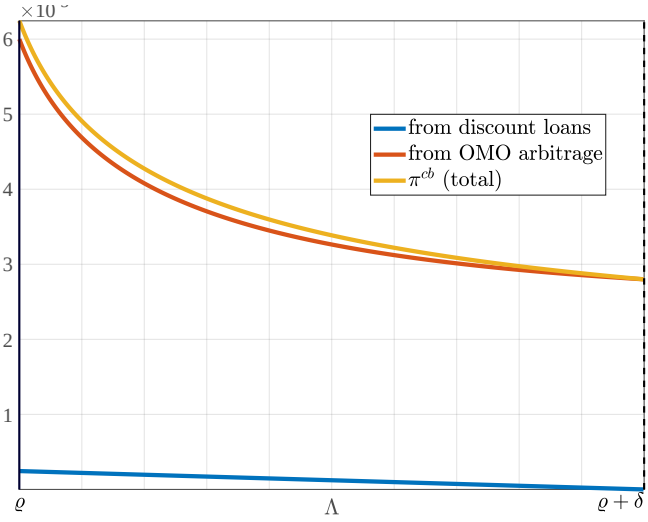
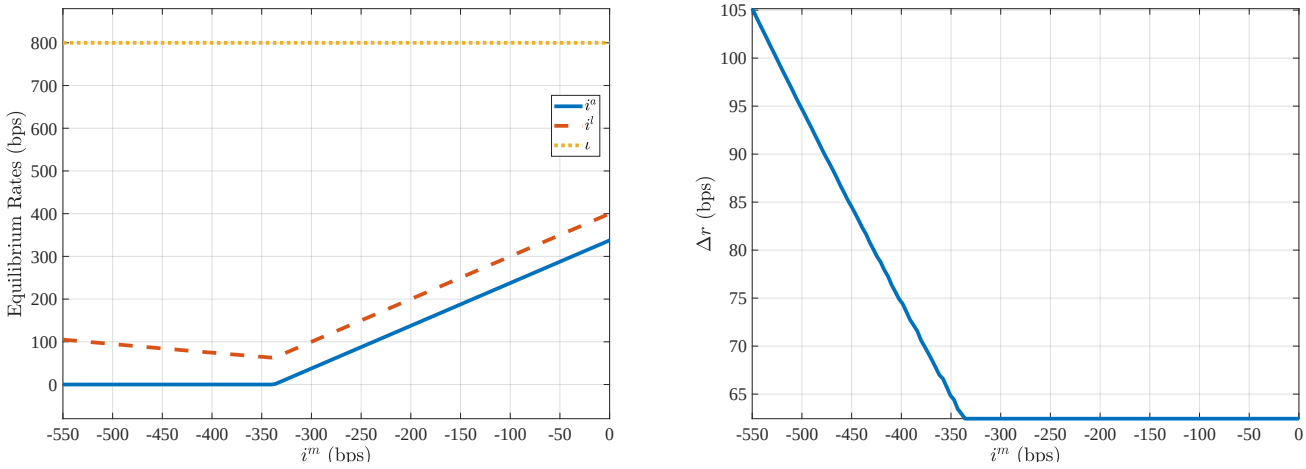


Figure 15: Composition of CB profit margins given Λ

Note: This figure plots the components of CB's profits over deposits as a function of liquidity ratio.

B.3 Additional Implementation Figures: Spread and Negative Interest on Reserves

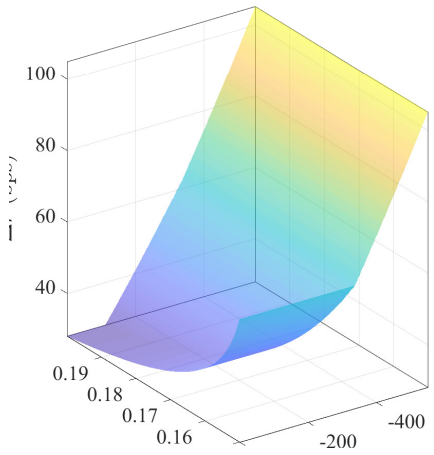


(a) Equilibrium Rates

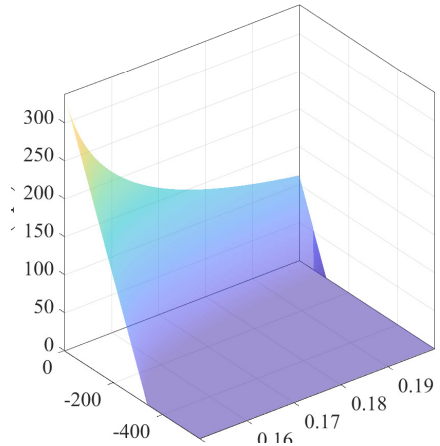
(b) Equilibrium Spread

Figure 16: Negative Interest on Reserves and the DZLB.

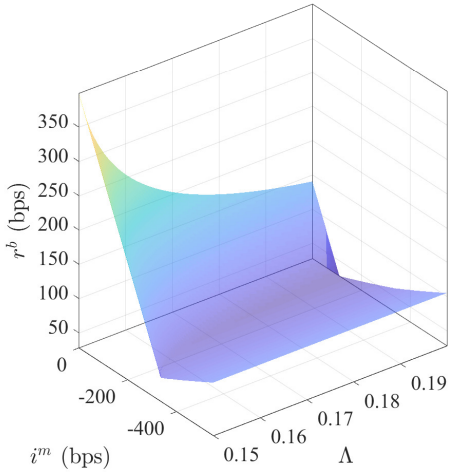
Note: This figure depicts the equilibrium rates and spread as a function of interest on reserves under DZLB. All the rates and spread are expressed in basis points.



(a) Equilibrium Spread



(b) Equilibrium Deposit Rate

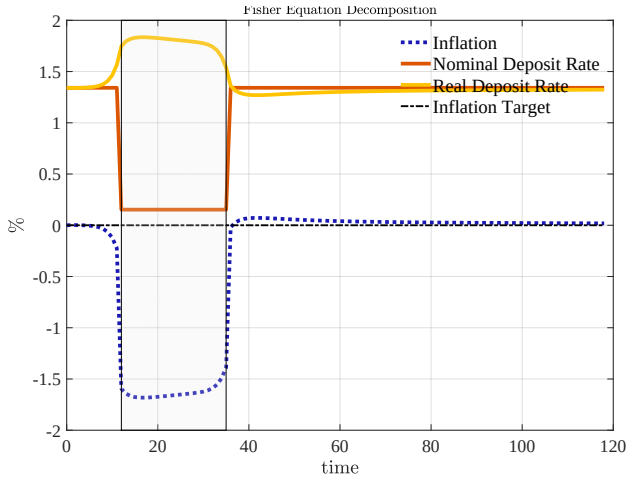


(c) Equilibrium Loan Rate

Figure 17: Negative Interest on Reserves, Liquidity Ratio and the DZLB.

Note: This figure presents the equilibrium spread, deposit rate and loan rate as functions of liquidity ratio and interest on reserves under DZLB. All the rates and spread are expressed in basis points.

B.4 Fisher Equation Decomposition



(a) Decomposition of Fisher Equation

Figure 18: Transition Dynamics of Fisher Equation Components under the Implementation of a Spread Reduction via OMO

Note: This figure reports the responses of inflation, nominal deposit rate, real deposit rate and inflation target according to Fisher equation decomposition, after the credit spread reduction implementation via OMO in Section 4.2.

C Properties of Modified Taylor Rule

This section discusses the modified Taylor rule used in the draft. We need to work with a Taylor rule that allows for both, a short-term choice of how the policy rate reacts to the credit crunch, but also respects long-run zero inflation, and the Taylor principle. For that we specify the following rule:

$$i_t^m = \bar{i}_t^m + \eta_t \cdot (\pi_t - \pi_{ss})$$

where η_t is time-varying. The path $\bar{i}_t^m$ is a path for a discretionary rate that satisfies:

$$\bar{i}_t^m = i_\infty^m + \left(\bar{i}_0^m - i_\infty^m\right) \cdot \exp\left(-\zeta^{LR} t\right) + \left(i_{0^-}^m - \bar{i}_0^m\right) \cdot \exp\left(-\zeta^{SR} t\right).$$

In this Taylor rule, the value i_∞^m is chose to guarantee an inflation target π_{ss}. The term $\bar{i}_d^m$ captures the attraction point of the policy rate chosen by the CB upon a shock. The rate $i_{t^-}^m$ is the policy rate, the instant before a shock. Finally, the term $\exp\left(-\zeta_t^{SR} t\right)$ captures a degree of responsiveness to the shock: the speed at which the discretionary policy kicks, whereas $\exp\left(-\zeta_t^{LR} t\right)$ the speed of reversal of the discretionary policy, to the long-run target. In what follows we assume $\zeta^{LR} < \zeta^{SR}$. This choice has several desirable properties:

1. First, observe that for any finite pair $\left\{\zeta^{SR}, \zeta^{LR}\right\}$ we have the following:

   $$\lim_{t\to\infty} \bar{i}_t^m = i_\infty^m.$$

2. Consider that for any finite pair $\left\{\zeta^{SR}, \zeta^{LR}\right\}$ we have the following:

   $$\lim_{t\to 0^+} \bar{i}_t^m = i_{0^-}^m.$$

3. Consider that for any finite pair $\left\{\zeta^{LR}\right\}$ we have the following:

   $$\lim_{t\to 0^+} \lim_{\zeta^{SR}\to\infty} \bar{i}_t^m = \bar{i}_0^m,$$

   meaning that the adjustment is immediate.

4. Consider that for any finite pair $\left\{\zeta^{SR}\right\}$ we have the following:

   $$\lim_{t\to\infty} \lim_{\zeta^{LR}\to\infty} \bar{i}_t^m = \bar{i}_0^m,$$

   meaning that the attraction point is the discretionary point.

5. Consider the limit, $\zeta^{LR}/\zeta^{SR} \to \infty$, then speed of responsiveness is immediate and

   $$\begin{aligned}
   \lim_{t\to\infty} \lim_{\zeta^{SR}/\zeta^{LR}\to\infty} \bar{i}_t^m &= \lim_{t\to\infty} \lim_{\zeta^{SR}/\zeta^{LR}\to\infty} i_\infty^m + \exp\left(-\zeta^{SR} t\right) \left[\left(\bar{i}_0^m - i_\infty^m\right) \cdot \exp\left(-\left(\zeta^{LR} - \zeta^{SR}\right) t\right) + \left(i_{0^-}^m - \bar{i}_0^m\right)\right] \\
   &= i_\infty^m + \left(\bar{i}_0^m - i_\infty^m\right) \lim_{t\to\infty} \lim_{\zeta^{SR}/\zeta^{LR}\to\infty} \exp\left(-\zeta^{SR} t\right) \left[\exp\left(-\left(\zeta^{LR} - \zeta^{SR}\right) t\right)\right] \\
   &= i_\infty^m + \left(\bar{i}_0^m - i_\infty^m\right) \lim_{t\to\infty} \lim_{\zeta^{SR}/\zeta^{LR}\to\infty} \exp\left(-\zeta^{SR} t\right) \left[\exp\left(\zeta^{SR}\left(1 - \zeta^{LR}/\zeta^{SR}\right)\right) t\right] \\
   &= \bar{i}_0^m.
   \end{aligned}$$

   Where the last line follows by L'Hospital rule.

6. Monotonicity of $\bar{i}_t^m$. Let's assume $i_\infty^m > \bar{i}_0^m$ and $i_{0^-}^m > \bar{i}_0^m$, which is the scenario in our simulation.

   If $\zeta^{LR} < \zeta^{SR}$, then

$$\frac{\partial \bar{i}_t^m}{\partial t} \gtreqless 0 \text{ iff } t \gtreqless \frac{1}{\zeta^{SR} - \zeta^{LR}} \ln\left(\frac{\zeta^{SR}}{\zeta^{LR}} \cdot \frac{i_{0^-}^m - \bar{i}_0^m}{i_\infty^m - \bar{i}_0^m}\right).$$

   This means the path of $\bar{i}_t^m$ first decreases over time from $i_{0^-}^m$, then increases back to i_∞^m, which motivates our choice. If instead, $\zeta^{LR} > \zeta^{SR}$, then

$$\frac{\partial \bar{i}_t^m}{\partial t} \gtreqless 0 \text{ iff } t \lesseqgtr \frac{1}{\zeta^{LR} - \zeta^{SR}} \ln\left(\frac{\zeta^{LR}}{\zeta^{SR}} \cdot \frac{i_\infty^m - \bar{i}_0^m}{i_{0^-}^m - \bar{i}_0^m}\right).$$

   This means the path of $\bar{i}_t^m$ first increases over time from $i_{0^-}^m$, then decreases back to i_∞^m. Not a desired property.

Path of η_t. Upon a shock, the path of η_t satisfies:

$$\eta_t = \eta_{t^-} \cdot \exp\left(-\xi^{LR} t\right) + \eta_{ss}\left(1 - \exp\left(-\xi^{LR} t\right)\right)$$

We use the equation **??** as the path of η_t based on the following observation: To implement a nominal rate i_t^a, we have the following equation:

$$i_t^a = i_t^m + \frac{1}{2}\left[\chi_t^+ + (1-\delta)\chi_t^-\right].$$

Combining the above equation with the Taylor Rule 15 and the Fisher equation $i_t^a = r_t^a + \pi_t$, we can rewrite π_t and i_t^m as follows:

$$\pi_t = \pi_{ss} + \frac{1}{\eta_t - 1}\left\{r_t^a - \frac{1}{2}\left[\chi_t^+ + (1-\delta)\chi_t^-\right] + \pi_{ss} - \bar{i}_t^m\right\},$$

and

$$i_t^m = -\frac{1}{\eta_t - 1}\bar{i}_t^m + \frac{\eta_t}{\eta_t - 1}\left\{r_t^a - \frac{1}{2}\left[\chi_t^+ + (1-\delta)\chi_t^-\right] + \pi_{ss}\right\}.$$

Therefore, if $\eta_t > 1$, a reduction in the discretionary rate $\bar{i}_t^m$ directly increases both π_t and i_t^m. However, we want to obtain that a discretionary rate $\bar{i}_t^m$ directly increases π_t and decreases i_t^m. Moreover, if $\eta_t > 1$, we are not able to reduce i_t^m to DZLB by decreasing $\bar{i}_t^m$.

D Solution Algorithm

The computational method follows (Achdou et al., 2019) closely. The main differences are the presence of the net asset position and the spread. Propositions 1, **??** and **??** are the objects we need to solve the model. They allow us to solve the model entirely by solving for the equilibrium path of a single price. For example, we can solve the model by solving the path for a real deposit rate r_t^a. The spread Δr_t follows immediately from Proposition 1 if we know the path for ι_t and Λ_t set by the CB. The real spread gives us r_t^l. To solve the household's problem, we need the path for $\left\{r_t^a, r_t^l, T_t\right\}$. The path for T_t is must be consistent with (20) and this yields a path for real government liabilities, $\mathcal{E}_t$. Then, $\mathcal{E}_t$ together with the evolution of $f(s,t)$ obtained from the household's problem, yield two sides of one equation enters (21). The rate equilibrium rate r_t^a must be the one that solves (21) implicitly.

Note that given the real credit spread Δr and government's net-asset position $\mathcal{E}$, the HJB equation (**??**), KF equation (13) and the real market clearing condition (21) imply that the equilibrium solution to the real markets is independent of implementation variables. Thus we divide the solution algorithm into two parts: the part of real market and the part of implementation. For the part of real market, the path of credit spread is taken as given. For the part of implementation, we simply use the equations (8) in Proposition 1 to show that the target credit spread is within the range of our calibration. Our algorithm closely follows the finite difference in Achdou et al. (2017).

D.1 Solution Algorithm: Stationary Equilibrium in Real Markets

We need to compute the value of deposit rate that satisfies the real market clearing condition (21) in steady state. We use an iteration algorithm that proceeds as follows. First, we take the real credit spread Δr as given, consider an initial guess of deposit rate $r^{a,0}$, total output Y, and fiscal transfer T, and set the iteration index $j, l := 0$. Then:

1. **Individual household's problem.** Given $r^{a,l}$, $Y^{j,l}$ and $T^{j,l}$, solve the household's value function $V^{j,l}(s)$ from HJB equation (**??**) using a finite difference method. Calculate the consumption function $c^{j,l}(s)$ and production technology choice $u^{j,l}(s)$.

2. **Aggregate distribution.** Given $\mu^{j,l}(s)$ and $c^{j,l}(s)$, solve the KF equation (13) for $f^{j,l}(s)$ using a finite difference method.

3. **Fiscal transfer and total output.** Given $c^{j,l}(s)$, $f^{j,l}(s)$, calculate aggregate output

$$Y^{j+1,l} = \int_{\bar{s}}^{\infty} y\left(u^{j,l}(s)\right) f^{j,l}(s)\, ds$$

and fiscal transfer

$$T^{j+1,l} = r^{a,l} \cdot e_f \cdot \int_0^{\infty} s f^{j,l}(s)\, ds - \Delta r \cdot \int_{\bar{s}}^{0} s f^{j,l}(s)\, ds.$$

If $\left\{Y^{j+1,l}, T^{j+1,l}\right\}$ is close enough to $\left\{Y^{j,l}, T^{j,l}\right\}$, proceed to 4. Otherwise, set $j := j+1$ and proceed to 1.

4. **Equilibrium deposit rate.** Given $f^{j,l}(s)$, compute the net supply of real financial claims

$$S\left(r^{a,l}\right) = \int_{\bar{s}}^{\infty} s f^{j,l}(s)\, ds + e_f \cdot \int_0^{\infty} s f^{j,l}(s)\, ds$$

and update the interest rate: if $S\left(r^{a,l}\right) > 0$, decrease it to $r^{a,l+1} < r^{a,l}$ and vice versa. If $S\left(r^{a,l}\right)$ is close enough to 0, stop. Otherwise, set $l := l+1$ and $j = 0$, and proceed to 1.

D.1.1 Solution to the HJB equation

The household's HJB equation is solved using an upwind finite difference scheme similar to Achdou et al. (2017). It approximates the value function $V(s)$ on a finite grid with step $\Delta s : s \in \{s_1, ..., s_I\}$, where $s_i = s_{i-1} + \Delta s = s_1 + (i-1)\Delta s$ for $2 \leq i \leq I$. The bounds are $s_1 = \bar{s}$ and $s_N = s^{max}$, such that $\Delta s = (s^{max} - \bar{s}) / (I-1)$. The upper bound s^{max} is an arbitrarily large number such that $f(s,t) = 0$ for all $s > s^{max}$. We use the short-hand notation $V_i \equiv V(s_i)$, and similarly for the policy function u_i and c_i.

Note that the HJB involves the first and second derivatives of the value function, $V_i' = V'(s_i)$ and $V_i'' = V''(s_i)$. The first derivative is approximated with either a forward (F) or a backward (B) approximation,

$$V_i' \approx \partial_F V_i \equiv \frac{V_{i+1} - V_i}{\Delta s}, \tag{36}$$

$$V_i' \approx \partial_B V_i \equiv \frac{V_i - V_{i-1}}{\Delta s}. \tag{37}$$

The second-order derivative is approximated by a central difference:

$$V_i'' \approx \partial_{ss} V_i \equiv \frac{V_{i+1} - 2V_i + V_{i-1}}{(\Delta s)^2}. \tag{38}$$

Let the superscript n be the iteration counter. The HJB equation is approximated by the following upwind scheme,

$$\frac{V_i^{n+1} - V_i^n}{\Delta} + \rho V_i^{n+1} = U(c_i^n) + \partial_F V_i^{n+1} \cdot \left(\mu_{i,F}^n\right)^+ + \partial_B V_i^{n+1} \cdot \left(\mu_{i,B}^n\right)^- + \frac{1}{2}(\sigma_i^n)^2 \partial_{ss} V_i^{n+1}, \tag{39}$$

where

$$\mu_{i,F}^n = r(s_i) \cdot s_i - (\partial_F V_i^n)^{-1/\gamma} + y(u_i^n) + T, \tag{40}$$

$$\mu_{i,B}^n = r(s_i) \cdot s_i - (\partial_B V_i^n)^{-1/\gamma} + y(u_i^n) + T, \tag{41}$$

and $\left(\sigma_i^n\right)^2 = \sigma^2\left(u_i^n\right)$.

The optimal consumption is set to

$$c_i^n = (\partial V_i^n)^{-1/\gamma}, \tag{42}$$

where

$$\partial V_i^n = \partial_F V_i^n \mathbf{1}_{\mu_{i,F}^n > 0} + \partial_B V_i^n \mathbf{1}_{\mu_{i,B}^n < 0} + \partial \bar{V}_i^n \mathbf{1}_{\mu_{i,F}^n \leq 0} \mathbf{1}_{\mu_{i,B}^n \geq 0}.$$

In the above expression, $\partial \bar{V}_i^n = \left(\bar{c}_i^n\right)^{-\gamma}$ where $\bar{c}_i^n$ is the consumption level such that $\mu_i^n = 0$, i.e.,

$$\bar{c}_i^n = r(s_i) \cdot s_i + y(u_i^n) + T.$$

The choice of production technology u_i^n is such that $u_i^n = H$ if and only if

$$\begin{aligned} &U(c_i^n(H)) + \partial_F V_i^{n+1} \cdot \left(\mu_{i,F}^n(H)\right)^+ + \partial_B V_i^{n+1} \cdot \left(\mu_{i,B}^n(H)\right)^- + \frac{1}{2}(\sigma_i^n(H))^2 \partial_{ss} V_i^{n+1} \\ \leq &U(c_i^n(L)) + \partial_F V_i^{n+1} \cdot \left(\mu_{i,F}^n(L)\right)^+ + \partial_B V_i^{n+1} \cdot \left(\mu_{i,B}^n(L)\right)^-, \end{aligned} \tag{43}$$

where $c_i^n(H)$ denotes the optimal consumption choice given $u = H$, and the other variables are defined in a similar way.

Substituting the definition of the derivatives (36), (37) and (38), equation (39) is

$$\frac{V_i^{n+1}-V_i^n}{\Delta}+\rho V_i^{n+1}=U\left(c_i^n\right)+\frac{V_{i+1}^{n+1}-V_i^{n+1}}{\Delta s}\cdot\left(\mu_{i,F}^n\right)^+ +\frac{V_i^{n+1}-V_{i-1}^{n+1}}{\Delta s}\cdot\left(\mu_{i,B}^n\right)^- +\frac{1}{2}\left(\sigma_i^n\right)^2\frac{V_{i+1}^{n+1}-2V_i^{n+1}+V_{i-1}^{n+1}}{\left(\Delta s\right)^2}.$$

Collecting terms with the same subscripts on the right-hand side

$$\begin{cases}\frac{V_i^{n+1}-V_i^n}{\Delta}+\rho V_i^{n+1}=U\left(c_i^n\right)+\alpha_i^n V_{i-1}^{n+1}+\beta_i^n V_i^{n+1}+\zeta_i^n V_{i+1}^{n+1}\\ \alpha_i^n=-\frac{\left(\mu_{i,B}^n\right)^-}{\Delta s}+\frac{\left(\sigma_i^n\right)^2}{2(\Delta s)^2}\\ \beta_i^n=-\frac{\left(\mu_{i,F}^n\right)^+}{\Delta s}+\frac{\left(\mu_{i,B}^n\right)^-}{\Delta s}-\frac{\left(\sigma_i^n\right)^2}{(\Delta s)^2}\\ \zeta_i^n=\frac{\left(\mu_{i,F}^n\right)^+}{\Delta s}+\frac{\left(\sigma_i^n\right)^2}{2(\Delta s)^2}\end{cases}\tag{44}$$

Note that $\alpha_1=0$, and we set $\zeta_I=0$ for the stability of the algorithm. Equation (44) is a system of I linear equations which can be written in the following matrix form:

$$\frac{1}{\Delta}\left(\mathbf{V}^{n+1}-\mathbf{V}^n\right)+\rho\mathbf{V}^{n+1}=\mathbf{U}^n+\mathbf{A}^n\mathbf{V}^{n+1}$$

where

$$\mathbf{A}^n=\begin{bmatrix}\beta_1^n & \zeta_1^n & 0 & 0 & \cdots & 0\\ \alpha_2^n & \beta_2^n & \zeta_2^n & 0 & \cdots & 0\\ 0 & \alpha_3^n & \beta_3^n & \zeta_3^n & \cdots & 0\\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots\\ 0 & 0 & \ddots & \alpha_{I-1}^n & \beta_{I-1}^n & \zeta_{I-1}^n\\ 0 & 0 & \cdots & 0 & \alpha_I^n & \beta_I^n\end{bmatrix},\ \mathbf{V}^{n+1}=\begin{bmatrix}V_1^{n+1}\\ V_2^{n+1}\\ V_3^{n+1}\\ \vdots\\ V_{I-1}^{n+1}\\ V_I^{n+1}\end{bmatrix},\ \mathbf{U}^n=\begin{bmatrix}U\left(c_1^n\right)\\ U\left(c_2^n\right)\\ U\left(c_3^n\right)\\ \vdots\\ U\left(c_{I-1}^n\right)\\ U\left(c_I^n\right)\end{bmatrix}.\tag{45}$$

The system in turn can be written as

$$\mathbf{B}^n\mathbf{V}^{n+1}=\mathbf{d}^n\tag{46}$$

where $\mathbf{B}^n=\left(\frac{1}{\Delta}+\rho\right)\mathbf{I}-\mathbf{A}^n$ and $\mathbf{d}^n=\mathbf{U}^n+\frac{1}{\Delta}\mathbf{V}^n$.

The algorithm to solve the HJB is as follows. We take the interest rate $\{r\left(s_i\right)\}_{i=1}^I$, total output Y and fiscal transfer T as given and begin with an initial guess $\left\{V_i^0\right\}_{i=1}^I$. Set $n=0$. Then:

1. Compute $\left\{\partial_F V_i^n,\partial_B V_i^n\right\}_{i=1}^I$ using (36) and (37).
2. Compute $\left\{c_i^n,u_i^n\right\}_{i=1}^I$ using (42) and (43) and $\left\{\mu_{i,F}^n,\mu_{i,B}^n\right\}_{i=1}^I$ using (40) and (41).
3. Find $\left\{V_i^n\right\}_{i=1}^I$ solving the linear system of equations (46).
4. If $\left\{V_i^{n+1}\right\}$ is close enough to $\left\{V_i^n\right\}$, stop. Otherwise set $n:=n+1$ and proceed to step 1.

D.1.2 Solve KFE in Stationary Equilibrium

The stationary distribution of real wealth satisfies the Kolmogorov Forward equation:

$$0=-\frac{\partial}{\partial s}\left[\mu\left(s\right)f\left(s\right)\right]+\frac{1}{2}\frac{\partial^2}{\partial s^2}\left[\sigma_s^2\left(s\right)f\left(s\right)\right],\tag{47}$$

$$1 = \int_{\bar{s}}^{\infty} f(s)\, ds. \tag{48}$$

We also solve the equation using a finite difference scheme. We use the notation $f_i \equiv f(s_i)$ .The system can be expressed as

$$\begin{aligned} 0 = & -\frac{f_i \left(\mu_{i,F}^n\right)^+ - f_{i-1}\left(\mu_{i-1,F}^n\right)^+}{\Delta s} - \frac{f_{i+1}\left(\mu_{i+1,B}^n\right)^- - f_i\left(\mu_{i,B}^n\right)^-}{\Delta s} \\ & + \frac{1}{2}\frac{\left(\sigma_{i+1}^n\right)^2 f_{i+1} - 2\left(\sigma_i^n\right)^2 f_i + \left(\sigma_{i-1}^n\right)^2 f_{i-1}}{(\Delta s)^2}, \end{aligned}$$

or equivalently

$$f_{i-1}\zeta_{i-1} + f_i\beta_i + f_{i+1}\alpha_{i+1} = 0.$$

The linear equations system can be written as

$$\mathbf{A^T f} = \mathbf{0}, \tag{49}$$

where $\mathbf{A^T}$ is the transpose of $\mathbf{A} = \lim_{n\to\infty} \mathbf{A}^n$. Notice that $\mathbf{A}^n$ is the approximation of the operator $\mathcal{A}$ and $\mathbf{A^T}$ is the approximation of the adjoint operator $\mathcal{A}^*$. In order to impose the normalization constraint (48) we replace one of the entries of the zero vector in equation (49) by a positive constant. We solve the system (49) and obtain a solution $\hat{\mathbf{f}}$. Then we renormalize as

$$f_i = \frac{\hat{f}_i}{\sum_{i=1}^I \hat{f}_i \Delta s}.$$

The algorithm to solve the stationary distribution is as follows.

1. Given the interest rate $\{r(s_i)\}_{i=1}^I$, total output Y and fiscal transfer T, solve the HJB equation to obtain an estimate of the matrix $\mathbf{A}$.

2. Given $\mathbf{A}$ find the aggregate distribution $\mathbf{f}$.

D.2 Solution Algorithm: Transition Dynamics

The equilibrium transition path is solved in finite horizon $[0, T]$, assuming that the terminal state of the economy is steady state. We use an iterative algorithm as follows. Given the initial distribution of real wealth $f_0(s)$ and the path of exogenous shocks (e.g. equation (**??**) for a fiscal transfer shock, or the path of real credit spread Δr_t), guess a function $r_t^{a,0}$, total output Y_t, and fiscal transfer T_t, and set the iteration index $j, l := 0$. Then

0. **The asymptotic steady state.** The asymptotic steady-state value function and real wealth distribution are calculated from Section D.1.

1. **Individual household's problem.** Given $r_t^{a,l}$, $Y_t^{j,l}$ and $T_t^{j,l}$, and the terminal condition $V^{j,l}(s,T) = V_{ss}(s)$, solve the HJB equation (**??**) backward in time to compute the path of $V^{j,l}(s,t)$. Calculate the production technology choice $u^{j,l}(s,t)$ and consumption policy function $c^{j,l}(s,t)$.

2. **Aggregate distribution.** Given $c^{j,l}(s,t)$ and $u^{j,l}(s,t)$, solve the Kolmogorov Forward equation (13) with initial condition $f^{j,l}(s,0) = f_0(s)$ forward in time to compute the path for $f^{j,l}(s,t)$.

3. **Fiscal transfer and total output.** Given $c^{j,l}(s,t)$, $f^{j,l}(s,t)$, calculate the path of aggregate output,

$$Y_t^{j+1,l} = \int_{\bar{s}}^{\infty} y\left(u^{j,l}(s,t)\right) f^{j,l}(s,t)\, ds,$$

and the path of fiscal transfer

$$T_t^{j+1,l} = r_t^{a,l} \cdot \mathcal{E}_t - \Delta r_t \cdot \int_{\bar{s}}^{0} s f^{j,l}(s,t)\, ds. If \left\{Y_t^{j+1,l}, T_t^{j+1,l}\right\}_{t-0}^{T}$$

is close enough to $\left\{Y_t^{j,l}, T_t^{j,l}\right\}_{t=0}^{T}$, proceed to 4. Otherwise, set $j := j+1$ and proceed to 1.

4. **Equilibrium deposit rate.** Given $f^{j,l}(s,t)$, calculate

$$S\left(r_t^{a,l}, t\right) = \int_{\bar{s}}^{\infty} s f^{j,l}(s,t)\, ds + \mathcal{E}_t$$

and update $r_t^{a,l+1} = r_t^{a,l} - \xi \frac{\partial S\left(r_t^{a,l},t\right)}{\partial t}$ for each t, where $\xi > 0$ is a parameter of update. If $\max_t \left\{\left|S\left(r_t^{a,l},t\right)\right|\right\}$ is close enough to 0, stop. Otherwise, set $l := l+1$ and $j = 0$, and proceed to 1.

D.2.1 Solution to the HJB Equation

The dynamic HJB equation (**??**) can be approximated using an upwind scheme as

$$\rho \mathbf{V}^n = \mathbf{U}^{n+1} + \mathbf{A}^{n+1}\mathbf{V}^n + \frac{1}{\Delta t}\left(\mathbf{V}^{n+1} - \mathbf{V}^n\right),$$

where $\mathbf{A}^{n+1}$ is defined in an analogous fashion to (45), and $\Delta t = T/N$ denotes the time length of each discrete period. We start with the terminal condition $\mathbf{V}^N = \mathbf{V}_{ss}$ and solve the path of value function backward, where $\mathbf{V}_{ss}$ denote the solution to stationary equilibrium obtained from Section D.1. For each $n = 0, 1, ..., N-1$, define $\mathbf{B}^n = \left(\frac{1}{\Delta t} + \rho\right)\mathbf{I} - \mathbf{A}^{n+1}$ and $\mathbf{d}^{n+1} = \mathbf{U}^{n+1} + \frac{1}{\Delta}\mathbf{V}^{n+1}$,and we can solve

$$\mathbf{V}^n = \left(\mathbf{B}^n\right)^{-1}\mathbf{d}^{n+1}.$$

D.2.2 Solution to the KF Equation

Let $\{\mathbf{A}^n\}_{n=1}^{N-1}$ be the solution obtained from Section D.2.1. It is the approximation to the operator $\mathcal{A}$. Using a finite difference scheme similar to the one we employed in Section D.1.2, we obtain:

$$\frac{\mathbf{f}^{n+1} - \mathbf{f}^n}{\Delta t} = \left(\mathbf{A}^n\right)^{\mathbf{T}}\mathbf{f}^{n+1},$$

which implies

$$\mathbf{f}^{n+1} = \left(\mathbf{I} - \Delta t\left(\mathbf{A}^n\right)^{\mathbf{T}}\right)^{-1}\mathbf{f}^n,\ n = 0, 1, ..., N-1. \tag{50}$$

We start from the initial period condition $\mathbf{f}^0 = \mathbf{f}_0$ and solve the KFE forward using (50).

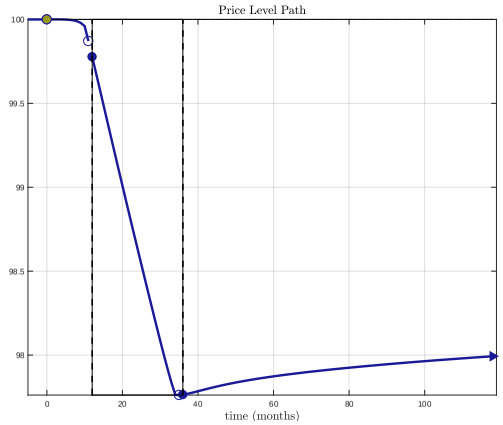
E Supplementary Section - Alternative Implementations

Components of Fisher Equation - Implementation of a Spread via OMO. Figure 18 shows the decomposition of inflation and the real and nominal deposit rates produced by the implementation of the spread in Figure **??**. The increase in real rates follows from the dynamics of the real credit spread. Deposit rates are constant until the OMO is actually carried out. Inflation follows the difference between both paths. The rate on reserves is set to implement a zero inflation target in the long-run.

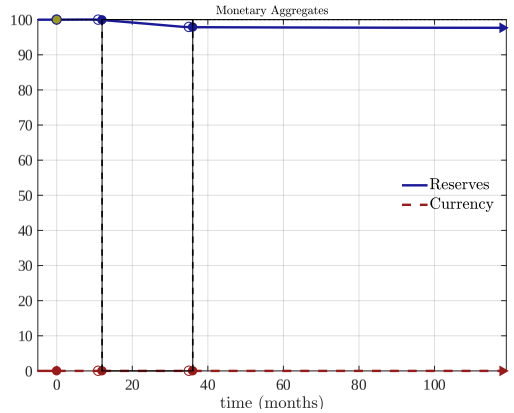
Implementation of a Spread via Reduction in Corridor Rates. Figure 19 describes the details of a reduction in ι that implements the same spread as the OMO in Figure **??**. The figure also reports the decomposition in Figure 18. The qualitative pattern is almost identical, although the quantities are not the same. Since real spreads are independent of inflation, the real deposit rate is the same. However, the nominal deposit rate decreases by slightly more than with an OMO. Notice how in Panel (b) there is no increase in the quantity of reserves.

Implementation of a Spread via Increase of i^m at the DZLB. Figure 20 describes the details of an increase in i^m that implements the same spread as the OMO in Figure **??** and the reduction in ι in Figure 19. The qualitative pattern is now different. First, for the implementation to work at all, the economy must be at the DZLB, because only in this region do changes in i^m that keep ι constant have real effects. At the DZLB currency holdings—Panel (b)—are positive. Since i^m is negative, but deposit rates are positive, as in the previous example, this implementation features steady-state deflation.

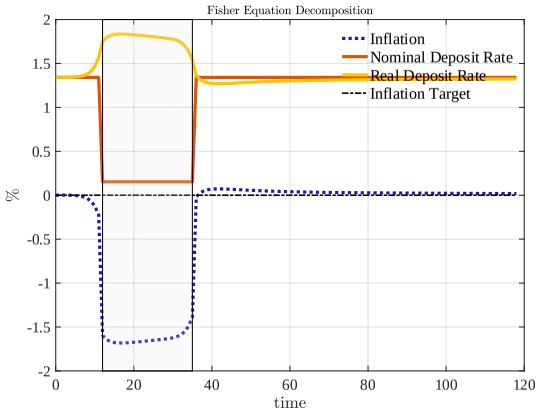
Here, the increase in the interest on reserves, once on negative territory, also produce a deflation since the pattern for real rates is the same. Different from the previous examples, at the DZLB, the deposit rate is flat at zero. The increase in interest on reserves reduces the loans rate, because it acts like a reduction the tax-like effect of negative reserve rates. We see also that the currency ratio of the economy falls.



(a) Price Index



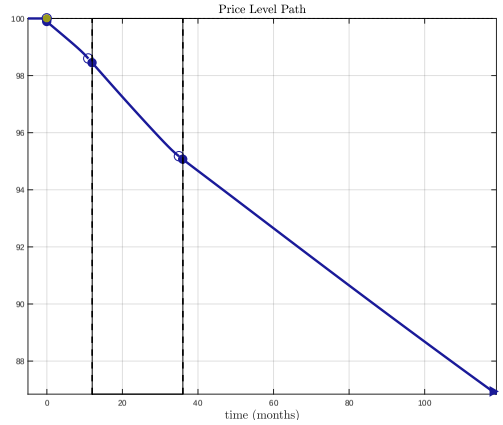
(b) Currency and Reserve Holdings



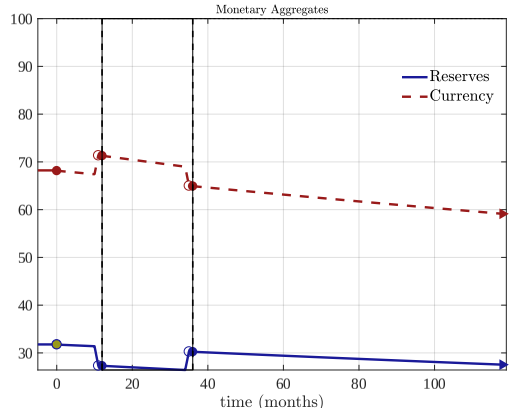
(c) Decomposition of Fisher Equation

Figure 19: Transition Dynamics of Fisher Equation Components under the Implementation of a Spread Reduction via Reduction in Corridor Rates

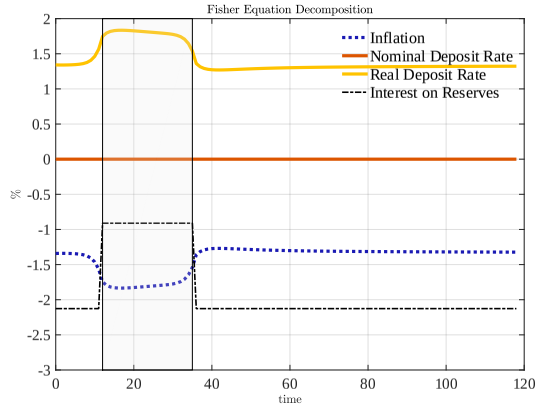
Note: This figure reports the responses of price index, monetary aggregates and Fisher equation components, after a reduction in corridor rate ι that implements the same spread reduction as the OMO in Figure **??**.



(a) Price Index



(b) Currency and Reserve Holdings



(c) Decomposition of Fisher Equation

Figure 20: Transition Dynamics of Fisher Equation Components under the Implementation of a Spread Reduction via Increase of Interest on Reserves

Note: This figure reports the responses of price index, monetary aggregates and Fisher equation components, after an increase in i^m that implements the same spread reduction as the OMO in Figure **??**.

F Proofs

F.1 Proof of Proposition 1

The individual bank takes $\{\Lambda, \chi^+, \chi^-\}$ and all rates as given. Consider an individual bank's problem:

$$\pi^b = \max_{\{l,m,a\} \in \mathbb{R}^3_+} \left(i^l l + i^m m - i^a a + \mathbb{E}\left[\chi\left(b; \theta, \iota\right)\right] \right)$$

subject to the budget constraint $l + m = a$ and the law of motion for reserve balances:

$$b\left(a, m\right) = \begin{cases} m \text{ with probability } 1/2 \\ m - \delta \cdot a \text{ with probability } 1/2 \end{cases}.$$

If we substitute out l from the budget constraint, the objective function becomes

$$\pi\left(m, a\right) \equiv \left(\left(i^m - i^l \right) \cdot m + \left(i^l - i^a \right) \cdot a + \frac{1}{2}\chi^+ m + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{[\frac{m}{a} > \delta]} + \chi^- \cdot \mathbb{I}_{[\frac{m}{a} \leq \delta]} \right)\left(m - \delta \cdot a\right) \right).$$

This problem is piece-wise linear. Let's consider the derivatives of the objective function, away from $m/a = \delta$. The derivatives of the objective with respect to the portfolio choices $\{m, a\}$ are

$$\frac{\partial \pi\left(m, a\right)}{\partial m} = \left(i^m - i^l \right) + \frac{1}{2}\chi^+ + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{[\frac{m}{a} > \delta]} + \chi^- \cdot \mathbb{I}_{[\frac{m}{a} \leq \delta]} \right)$$

and

$$\frac{\partial \pi\left(m, a\right)}{\partial m} = \left(i^l - i^a \right) - \frac{\delta}{2}\left(\chi^+ \cdot \mathbb{I}_{[\frac{m}{a} > \delta]} + \chi^- \cdot \mathbb{I}_{[\frac{m}{a} \leq \delta]} \right).$$

We know that $\chi^- > \chi^+$ if $\Lambda < \delta$ and that $\chi^- = \chi^+ = 0$ if $\Lambda \geq \delta$. We break the proof into two cases, depending on this condition.

- Assume that $\Lambda < \delta$ at the aggregate level. Then, $\left(i^m - i^l \right) + \chi^+ < 0$,because otherwise, $\frac{\partial \pi(m,a)}{\partial m} > 0$, for any a, and this means that banks demand infinite reserve balances. Similarly, it must be that $\left(i^m - i^l \right) + \frac{1}{2}\left(\chi^+ + \chi^- \right) \geq 0$, because otherwise, banks would demand no real balances in equilibrium. In particular, $\left(i^m - i^l \right) + \frac{1}{2}\left(\chi^+ + \chi^- \right) = 0$, because if the inequality is strict, banks would demand reserves balances equal to $m/a = \delta$, but at the aggregate level, this would contradict the condition that $\Lambda < \delta$. Similarly, it must be that $\left(i^l - i^a \right) > \frac{\delta}{2}\chi^+$, for otherwise banks would not issue deposits. Then, to guarantee a finite amount of deposits, given m, $\left(i^l - i^a \right) \leq \frac{\delta}{2}\chi^-$. In particular, it must be that $\left(i^l - i^a \right) \leq \frac{\delta}{2}\chi^-$, because if the inequality were strict, banks would decrease there issuance of deposits up to any point where $m/a > \delta$.
- Assume that $\Lambda \geq \delta$ at the aggregate level. Then, the problem becomes linear, as opposed to piece-wise linear. Since the objective is linear, then, it must be that banks earn zero profits, and that requires $i^m = i^l = i^a$.

Thus, taken together, these observations imply that a necessary condition for a positive and finite supply of loans and deposits are conditions (6) and (**??**). Since in equilibrium the demand of deposits and loans is finite, the result follows. Once we substitute (6) and (**??**), we obtain that banks earns zero expected profits from any choice of $\{a, m, l\}$. **QED.**

F.2 Proof of Proposition ??

1. We first prove that if (21) holds, then the goods market clears, which verifies Walras's law for a continuous time setting. Observe that if condition (21) holds, then taking time derivatives we obtain:

$$0 = \frac{\partial}{\partial t}\left[\int_{\bar{s}}^{\infty} sf(s,t)ds\right] + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right],$$

Then, we have:

$$0 = \int_{\bar{s}}^{\infty} s\frac{\partial}{\partial t}\left[f(s,t)\right]ds + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right],$$

but recall that if the KFE equation holds, then:

$$0 \quad = \quad \int_{\bar{s}}^{\infty} s\left[-\frac{\partial}{\partial s}\left[\mu\left(s,t\right)f\left(s,t\right)\right] + \frac{1}{2}\frac{\partial^2}{\partial s^2}\left[\sigma_s^2\left(s,t\right)f\left(s,t\right)\right]\right]ds + \frac{\partial}{\partial t}\left[\mathcal{E}_t\right].$$

Now, observe that, if we employ the integration by parts formula:

$$-\int_{\bar{s}}^{\infty} s\frac{\partial}{\partial s}\left[\mu\left(s,t\right)f\left(s,t\right)\right]ds = -s\mu\left(s,t\right)f\left(s,t\right)|_{\bar{s}}^{\infty} + \int_{\bar{s}}^{\infty}\mu\left(s,t\right)f\left(s,t\right)ds.$$

We know that

$$-s\mu\left(s,t\right)f\left(s,t\right)|_{\bar{s}}^{\infty} = 0$$

and that

$$\int_{\bar{s}}^{\infty}\mu\left(s,t\right)f\left(s,t\right)ds = \int_{\bar{s}}^{\infty}\left[r_t\left(s\right)\left(s - m^h\left(s,t\right)/P_t\right) - \dot{P}_t/P_t\cdot m^h\left(s,t\right)/P_t - c\left(s,t\right) + h\left(u\left(s,t\right),t\right)\right]f\left(s,t\right)ds.$$

First, note that:

$$\int_{\bar{s}}^{\infty} r_t\left(s\right)sf\left(s,t\right)ds = \int_{\bar{s}}^{\infty} r_t^l\cdot sf\left(s,t\right)ds - \int_0^{\infty}\Delta r_t\cdot sf\left(s,t\right)ds.$$

Second, the household's problem solution implies $i_t^a\cdot m^h\left(s,t\right) = 0$ for any (s,t), and $m^h\left(s,t\right) = 0$ for any $s \leq 0$. Then we have

$$\begin{aligned}
&\int_{\bar{s}}^{\infty}\left(r_t\left(s\right) + \dot{P}_t/P_t\right)\left(m^h\left(s,t\right)/P_t\right)f\left(s,t\right)ds \\
=&i_t^l\int_{\bar{s}}^{0}\frac{m^h\left(s,t\right)}{P_t}f\left(s,t\right)ds + i_t^a\int_0^{\infty}\frac{m^h\left(s,t\right)}{P_t}f\left(s,t\right)ds \\
=&0.
\end{aligned}$$

Third, by definition,

$$\int_{\bar{s}}^{\infty}\left(-c\left(s,t\right) + h\left(u\left(s,t\right),t\right)\right)f\left(s,t\right)ds = Y_t - C_t + T_t.$$

Finally, the term:

$$\frac{1}{2}\int_{\bar{s}}^{\infty} s\cdot\frac{\partial^2}{\partial s^2}\left[\sigma_s^2(s,t)\,f(s,t)\right]ds=\frac{1}{2}\,s\cdot\frac{\partial}{\partial s}\left[\sigma_s^2(s,t)\,f(s,t)\right]\Big|_{\bar{s}}^{\infty}-\frac{1}{2}\int_{\bar{s}}^{\infty}\frac{\partial}{\partial s}\left[\sigma_s^2(s,t)\,f(s,t)\right]ds$$
$$=0-\frac{1}{2}\,\sigma_s^2(s,t)\,f(s,t)\Big|_{\bar{s}}^{\infty}=0.$$

Thus, we are left with:

$$r_t^l\int_{\bar{s}}^{\infty} sf(s,t)\,ds-\Delta r_t\int_0^{\infty} sf(s,t)\,ds+Y_t-C_t+T_t+\frac{\partial}{\partial t}\left[\mathcal{E}_t\right]=0.$$

But then, given the law of motion for real equity (20),

$$\frac{\partial}{\partial t}\left[\mathcal{E}_t\right]+r_t^l\int_{\bar{s}}^{\infty} sf(s,t)\,ds-\Delta r_t\int_0^{\infty} sf(s,t)\,ds+T_t=0.$$

This implies the goods market clearing condition.

2. Next, we proof that if (21) holds, the deposit and loans market must clear. The accounting identities in Section 2.2 and Lemma 1, show that if all markets clear, the real market clears. Then, by dividing (23) by the price level, we obtain:

$$-\int_{\bar{s}}^{0} sf(s,t)ds=\int_0^{\infty} sf(s,t)ds+\mathcal{E}_t,\ \text{for } t\in[0,\infty).$$

The proposition establishes that if this condition holds, all asset markets clear. To proceed with the proof, argue that if the condition holds, but one of the markets doesn't clear, we reach contradiction.

To see that, observe that real household's assets position equations (28) and (29), and (23) imply

$$\int_{\bar{s}}^{0} l_t^h(s)f(s,t)ds=\int_0^{\infty} a_t^h(s)f(s,t)ds+M0_t+L_t^f-M_t,\ \text{for } t\in[0,\infty).\tag{51}$$

Re-arranging terms leads, and using the money-market clearing condition, we obtain:

$$M_t^b+\int_{\bar{s}}^{0} l_t^h(s)f(s,t)ds-L_t^f=\int_0^{\infty} a_t^h(s)f(s,t)ds.$$

Now, recall that $M_t^b=-L_t^b+A_t^d$. Thus,

$$\left(\int_{\bar{s}}^{0} l_t^h(s)f(s,t)ds-L_t^f-L_t^b\right)=\int_0^{\infty} a_t^h(s)f(s,t)ds-A_t^d.$$

This equation guarantees that if there is no clearing in the loans market, there is no clearing in the deposit market by that same amount. Assume there is a deviation from market clearing in the amount ε. Then, an income $\Delta r\cdot\varepsilon$ would not be accounted. However, since all the spread is earned by the CB, it must be that $\varepsilon=0$. **QED.**

F.3 Proof of Proposition ??

The nominal profits of the CB are given by:

$$\pi_t^f=i_t^l L_t^f-i_t^m\left(M_t-M0_t\right)+\iota_t\left(1-\psi_t^-\right)B_t^-.$$

Note that the earnings from discount-window loans equal the average payment in the interbank market, and thus:

$$\iota_t \left(1 - \psi_t^-\right) B_t^- = -\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right]. \tag{52}$$

By Proposition 1, banks earn zero profits in expectation. Thus,

$$-\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right] = i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \tag{53}$$

Thus, substituting (52) and (53) into the expression for π_t^f above yields:

$$\begin{aligned}\pi_t^f &= i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \\ &= i_t^l L_t^h - i_t^a A_t^h,\end{aligned}$$

where we used the clearing condition in the money market, $M_t^b + M_t^0 = M_t$, the deposit market, $A_t^b = A_t^h$, and the loans market, $L_t^h = L_t^b + L_t^f$. Now, observe that:

$$\pi_t^f = -i_t^l P_t \int_{\bar{s}}^0 sf(s,t)ds - i_t^a \left(P_t \int_0^\infty sf(s,t)ds - M0_t.\right),$$

but we know from the household's problem that $i_t^a M0_t$=0. Hence, profits are given by:

$$\pi_t^f = -i_t^l P_t \int_{\bar{s}}^0 sf(s,t)ds - i_t^a P_t \int_0^\infty sf(s,t)ds.$$

Divide (23) by the price level to obtain:

$$-\int_{\bar{s}}^0 sf(s,t)ds = \int_0^\infty sf(s,t)ds + \mathcal{E}_t.$$

and thus:

$$\pi_t^f = \left(i_t^l - i_t^a\right) P_t \int_0^\infty sf(s,t)ds + i_t^l E_t = \Delta r_t P_t \int_0^\infty sf(s,t)ds + i_t^l E_t.$$

Dividing both sides by the price level leads to:

$$\frac{\pi_t^f}{P_t} = \Delta r_t \int_0^\infty sf(s,t)ds + i_t^l \mathcal{E}_t = \Delta r_t \int_0^\infty sf(s,t)ds + \left(r_t^a + \Delta r_t + \frac{\dot{P}_t}{P_t}\right)\mathcal{E}_t. \tag{54}$$

Then, note that:

$$d\mathcal{E}_t = \frac{dE_t}{P_t} - \frac{\dot{P}_t}{P_t}\mathcal{E}_t = \frac{\pi_t^f}{P_t} - T_t - \frac{\dot{P}_t}{P_t}\mathcal{E}_t.$$

But, a substitution of (54) yields:

$$d\mathcal{E}_t = \left((r_t^a + \Delta r_t)\mathcal{E}_t + \Delta r_t \int_0^\infty sf(s,t)ds - T_t\right)dt.$$

This proves Proposition **??**. **QED.**

F.4 Proof of Proposition 5

It suffices to show the equations for real credit spread and inflation rate. Along an equilibrium path for $\{r_t^a, \mathcal{E}_t, f_t, \Delta r_t, T_t\}$ the set of implementable nominal interbank rates and inflation rates is the set of $\{\dot{P}_t/P_t, \bar{i}_t^f\}$ where

$$\frac{\dot{P}_t}{P_t} = i_t^l - (\Delta r_t + r_t^a) = i_t^m + \frac{1}{2}\Big[\chi^+(\Lambda_t, \iota_t) + \chi^-(\Lambda_t, \iota_t)\Big] - \Delta r_t - r_t^a \tag{55}$$

$$\bar{i}_t^f = \chi^+(\Lambda_t, \iota_t)/\psi^+(\theta(\Lambda_t)) + i_t^m \tag{56}$$

for any $\{i_t^m, \iota_t, \mathcal{L}_t^f\}$ such that

$$\begin{aligned}
\Delta r_t &= r_t^l - r_t^a = i_t^l - \dot{P}_t/P_t - i_t^a + \dot{P}_t/P_t \\
&= \Delta i_t = \varrho\frac{\chi^+(\Lambda_t, \iota_t) + \chi^-(\Lambda_t, \iota_t)}{2} + \delta\frac{\chi^-(\Lambda_t, \iota_t) - \chi^+(\Lambda_t, \iota_t)}{2}, \\
\mathcal{L}_t^f &\leq -\int_{\bar{s}}^0 sf(s,t)ds, \quad (\iota_t, i_t^m) \in \mathbb{R}_+^2.
\end{aligned}$$

Equations (55) and (56) steams form definitions for nominal, real and interbank rate. The implementation constraint $\mathcal{L}_t^f \leq -\int_{\bar{s}}^0 sf(s,t)ds$ simply tells that there must be enough private liabilities to set $\mathcal{L}_t^f$. **QED.**

F.5 Proof of Corollary 2

It suffices to show that $\Delta r_t = 0$ when $i^m \geq 0$ and $\Lambda \geq \rho + \delta$. Note that the interbank market is satiated with reserves if $\Lambda_t \geq \bar{\Lambda} = \varrho + \delta$. Then the interbank market tightness is $\theta(\Lambda_t) = 0$ for any $\Lambda_t \geq \bar{\Lambda} = \varrho + \delta$. First, we must take the following limit

$$\lim_{\theta\to 0}\frac{\bar{\theta}(\theta)}{\theta} = \lim_{\theta\to 0}\frac{1}{\theta[1 + (\theta^{-1} - 1)\exp(\lambda)]} = \lim_{\theta\to 0}\frac{1}{\theta + (1-\theta)\exp(\lambda)} = \exp(-\lambda),$$

where $\bar{\theta}(\theta)$ is given by (33) in Appendix B. Then, given (η, λ), for any $\Lambda_t \geq \bar{\Lambda}$, (32) implies:

$$\begin{aligned}
\chi^+(\Lambda_t, \iota_t) &= \lim_{\theta\to 0}\iota_t\theta\left(\frac{\overline{\theta}(\theta)}{\theta}\right)^\eta\left(\frac{[\overline{\theta}(\theta)/\theta]^{1-\eta} - 1}{\overline{\theta}(\theta) - 1}\right) = 0, \\
\chi^-(\Lambda_t, \iota_t) &= \lim_{\theta\to 0}\iota_t\left(\frac{\overline{\theta}(\theta)}{\theta}\right)^\eta\left(\frac{\theta[\overline{\theta}(\theta)/\theta]^{1-\eta} - 1}{\overline{\theta}(\theta) - 1}\right) = \iota_t\exp(-\eta\lambda).
\end{aligned}$$

Although $\chi_t^- > 0$, there are not banks with reserves deficit, thus

$$\mathbb{E}\Big\{\chi_t[b(a, a-l)]|\theta_t\Big\} = \chi^+(\Lambda_t, \iota_t)\,(a - l - \varrho a) = 0$$

Hence, the bank's problem becomes

$$\pi_t^b = \max_{a,l}(i_t^l - i_t^m)l_t - (i_t^a - i_t^m)a_t$$

and by FOCs we obtain that $i_t^m = i_t^a = i_t^l = \bar{i}_t^f$. **QED.**

F.6 Proof of Proposition 2

1. [Corridor Regime] In this case, $\Lambda_t = \Lambda^{MB}\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right)$, $\theta(\Lambda_t) \in (0,1)$, $\left\{i^l, i^a, \Delta r\right\}$ is given by (6), (**??**) and (8), and $\{\chi^+, \chi^-\}$ is given by (3). Since $\Lambda^{MB}\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right)$ is increasing in $\mathcal{L}_t^f$, then the proof of Lemma **??** in Appendix **??** implies $\left\{\frac{\partial i^l}{\partial \mathcal{L}_t^f}, \frac{\partial i^a}{\partial \mathcal{L}_t^f}, \frac{\partial \Delta r}{\partial \mathcal{L}_t^f}\right\} < 0$. By (6), (**??**) and (8) one can observe that $\frac{\partial i^l}{\partial i_t^m} = \frac{\partial i^a}{\partial i_t^m} = 1$ and $\frac{\partial \Delta r}{\partial i_t^m} = 0$. By (3), both χ^+ and χ^- are proportional to ι, thus the elasticities of $\left\{i^l, i^m, \Delta r\right\}$ with respect to ι_t are all equal to 1.

[Floor Regime] In this case, $\theta(\Lambda_t) = 0$ and the proof of Corollary 2 establishes all the results.

[DZLB and negative i^m regime] In this case, the definition of Λ^{zlb} implies that $i^a \equiv 0$ and Λ_t is independent of $\mathcal{L}_t^f$. Thus $\mathcal{L}_t^f$ has no impact on $\left\{i^l, i^a, \Delta r\right\}$. The equilibrium $\left\{i^l, \Delta r\right\}$ are still given by (6) and (8). To prove the effects of $\{i_t^m, \iota_t\}$ on $\left\{i^l, \Delta r\right\}$, it suffices to show the sign of $\frac{\partial i^l}{\partial i^m}$ and $\frac{\partial i^l}{\partial \iota}$. We take total differentiation of (**??**). This gives

$$\begin{aligned} 0 =& di^m + \frac{1}{2}(1-\varrho+\delta)\left(\frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) \\ &+ \frac{1}{2}(1-\varrho-\delta)\left(\frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right), \end{aligned} \tag{57}$$

which implies

$$\left\{\frac{\partial \Lambda}{\partial i^m}, \frac{\partial \Lambda}{\partial \iota}\right\} > 0,$$

and

$$\begin{aligned} di^l &= di^m + \frac{1}{2}\left(\frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) + \frac{1}{2}\left(\frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) \\ &= \frac{\varrho-\delta}{2}\left(\frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) + \frac{\varrho+\delta}{2}\left(\frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right). \end{aligned}$$

Let $di^m > 0$ and $d\iota = 0$. Then by Lemma **??** we have

$$\frac{\partial i^l}{\partial i^m} = \frac{\varrho-\delta}{2} \cdot \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda}\frac{\partial \Lambda}{\partial i^m} + \frac{\varrho+\delta}{2} \cdot \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda}\frac{\partial \Lambda}{\partial i^m} < 0.$$

Let $di^m = 0$ and $d\iota > 0$. The proof of Lemma **??** in Appendix **??** and equation (3) imply that

$$\frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda} < \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda} < 0$$

and

$$\frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \iota} > \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \iota} > 0.$$

Then equation (57) implies

$$\begin{aligned} &\left(\frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^+(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) \\ =& -\frac{1-\varrho-\delta}{1-\varrho+\delta}\left(\frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \Lambda} d\Lambda + \frac{\partial \chi^-(\theta(\Lambda),\iota)}{\partial \iota} d\iota\right) \\ <& 0. \end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial i^l}{\partial \iota} &= \left(\frac{\varrho+\delta}{2} - \frac{1-\varrho-\delta}{1-\varrho+\delta}\frac{\varrho-\delta}{2}\right)\cdot\left(\frac{\partial \chi^{-}\left(\theta\left(\Lambda\right),\iota\right)}{\partial \Lambda}d\Lambda + \frac{\partial \chi^{-}\left(\theta\left(\Lambda\right),\iota\right)}{\partial \iota}d\iota\right)\\ &= \frac{\delta}{1-\rho+\delta}\cdot\left(\frac{\partial \chi^{-}\left(\theta\left(\Lambda\right),\iota\right)}{\partial \Lambda}d\Lambda + \frac{\partial \chi^{-}\left(\theta\left(\Lambda\right),\iota\right)}{\partial \iota}d\iota\right)\\ &> 0.\end{aligned}$$

This concludes the summary of the policy effects. **QED.**

F.7 Proof of Corollary 1

The discount window profits are equal to $\Delta r_t \int_0^\infty s f\left(s,t\right) ds$ since banks are competitive and earn zero profits. Given the same real credit spread Δr_t, the equilibrium real wealth distribution $f\left(s,t\right)$ is also same. Thus Corollary 1 is established. **QED.**

F.8 Proof of Corollary ??

The proof is established by change of variables. Note that in Problem 2 with $r_t^a = r_{ss}^a$, $\Delta r_t = 0$ and $T_{ss} = r_{ss}^a \mathcal{E}_{ss}$, the households' problem is

$$\begin{aligned}\rho V\left(s,t\right) &= \max_{\{c\}\geq 0, u\in\{L,H\}} U\left(c\right) + V_s\cdot\left(r_{ss}^a\cdot s - c + y\left(u\right) + T_{ss}\right) + \frac{1}{2}V_{ss}\sigma^2\left(u\right) + \dot{V}\\ &= \max_{\{c\}\geq 0, u\in\{L,H\}} U\left(c\right) + V_s\cdot\left(r_{ss}^a\cdot\left(s+\mathcal{E}_{ss}\right) - c + y\left(u\right)\right) + \frac{1}{2}V_{ss}\sigma^2\left(u\right) + \dot{V}\end{aligned}$$

subject to

$$s_t \geq \bar{s} \;\Leftrightarrow\; s_t + \mathcal{E}_{ss} \geq \bar{s} + \mathcal{E}_{ss}.$$

Denote $s_t^{(a)} = s_t + \mathcal{E}_{ss}$, $\bar{s}^{(a)} = \bar{s} + \mathcal{E}_{ss}$ and $V^{(a)}\left(s^{(a)},t\right) = V\left(s^{(a)} - \mathcal{E}_{ss}, t\right)$. Then the households' problem can be written as

$$\rho V^{(a)}\left(s_t^{(a)},t\right) = \max_{\{c\}\geq 0, u\in\{L,H\}} U\left(c\right) + V_{s^{(a)}}^{(a)}\cdot\left(r_{ss}^a\cdot s_t^{(a)} - c + y\left(u\right)\right) + \frac{1}{2}V_{s^{(a)}s^{(a)}}^{(a)}\sigma^2\left(u\right) + \dot{V}^{(a)}$$

subject to $s_t^{(a)} \geq \bar{s}^{(a)}$. This economy has the same equilibrium allocation as the original one. **QED.**

F.9 Proof of Proposition ??

The proof is similar to Corollary **??** and is also established by change of variables. Taking differentiation of $h\left(t\right)$ with respect to t gives us

$$0 = \dot{h}\left(t\right) = -T_t + r_t^a\cdot h\left(t\right),$$

which implies $T_t = r_t^a\cdot h\left(t\right)$. Note that a policy that sets $\iota_t = 0$ or satiate banks with reserves imply $\Delta r_t = 0$. Thus denote $s_t^{(a)} = s_t + h\left(t\right) \equiv s_t + \mathcal{E}_{ss}$, $\bar{s}_t^{(a)} = \bar{s} + h\left(t\right) \equiv \bar{s} + \mathcal{E}_{ss}$ and $V^{(a)}\left(s^{(a)},t\right) = V\left(s^{(a)} - \mathcal{E}_{ss}, t\right)$, the household's problem 2 with

$\Delta r_t = 0$ and $T_t = r_t^a \cdot h(t)$ can be written as

$$\rho V^{(a)}\left(s_t^{(a)}, t\right) = \max_{\{c\}\geq 0, u\in\{L,H\}} U(c) + V_{s^{(a)}}^{(a)} \cdot \left(r_t^a \cdot s_t^{(a)} - c + y(u)\right) + \frac{1}{2} V_{s^{(a)}s^{(a)}}^{(a)} \sigma^2(u) + \dot{V}^{(a)}$$

subject to$s_t^{(a)} \geq \bar{s}_t^{(a)}$. This economy has the same equilibrium allocation as the original one, and the allocation is independent of $\{T_t, \mathcal{E}_t\}$. **QED.**

G Additional Exercises

Effects of Changes in the IOR with under alternative Taylor rule. **The simulation results in this exercise are different from the previous exercise. In this exercise, the real variables (output, credit and job separation rate) are more responsive under a larger κ. However, in this exercise, a reduction in$\bar{i}_0^m$ always generates a reduction in the IOR i_t^m. In Figure 4 we consider the same policy shock to the discretionary component $\bar{i}_t^m$ as Figure 21, except that we use a new Taylor rule.**

Figure 21 compares the transition paths under three different values of κ, i.e. the sensitivity of inflation to unemployment rate in the Phillips curve: $\kappa = 0.0001, 0.1, 5$.

In Figure 21, we consider the exercise that deviates from a constant Taylor rule by inducing a change in the the discretionary component of the Taylor rule $\bar{i}_t^m$, see (xxx).

In this experiment the central bank sets the initial value $\bar{i}_0^m$ of the discretionary component $\bar{i}_t^m$, and controls the path of $\bar{i}_t^m$ as follows:

$$\bar{i}_t^m = i_\infty^m + \left(\bar{i}_0^m - i_\infty^m\right) \cdot \exp\left(-\bar{\zeta}^{LR} t\right) + \left(i_{0^-}^m - \bar{i}_0^m\right) \cdot \exp\left(-\bar{\zeta}^{SR} t\right).$$

Figure 21 compares the transition paths under three different values of κ, i.e. the sensitivity of inflation to unemployment rate in the Phillips curve: $\kappa = 0.0001, 0.1, 5$.

In the following figures, we consider the Taylor Rule such that $\eta_t \equiv \bar{\eta}$, $\zeta_t^{LR} = \bar{\zeta}^{LR}$ and $\zeta_t^{SR} = \bar{\zeta}^{SR}$, and simulate the paths under three different values of κ. This exercise is to articulate standard movement in response to an expansionary nominal policy rate change. The figures confirm the standard movement: with a higher value of κ, the real variables (output, credit and job separation rate) are less responsive, while the nominal variables (inflation rate) are more responsive. The only concern is that the policy change is a reduction in$\bar{i}_0^m$, but the IOR i_t^m increases when $\kappa = 5$. I

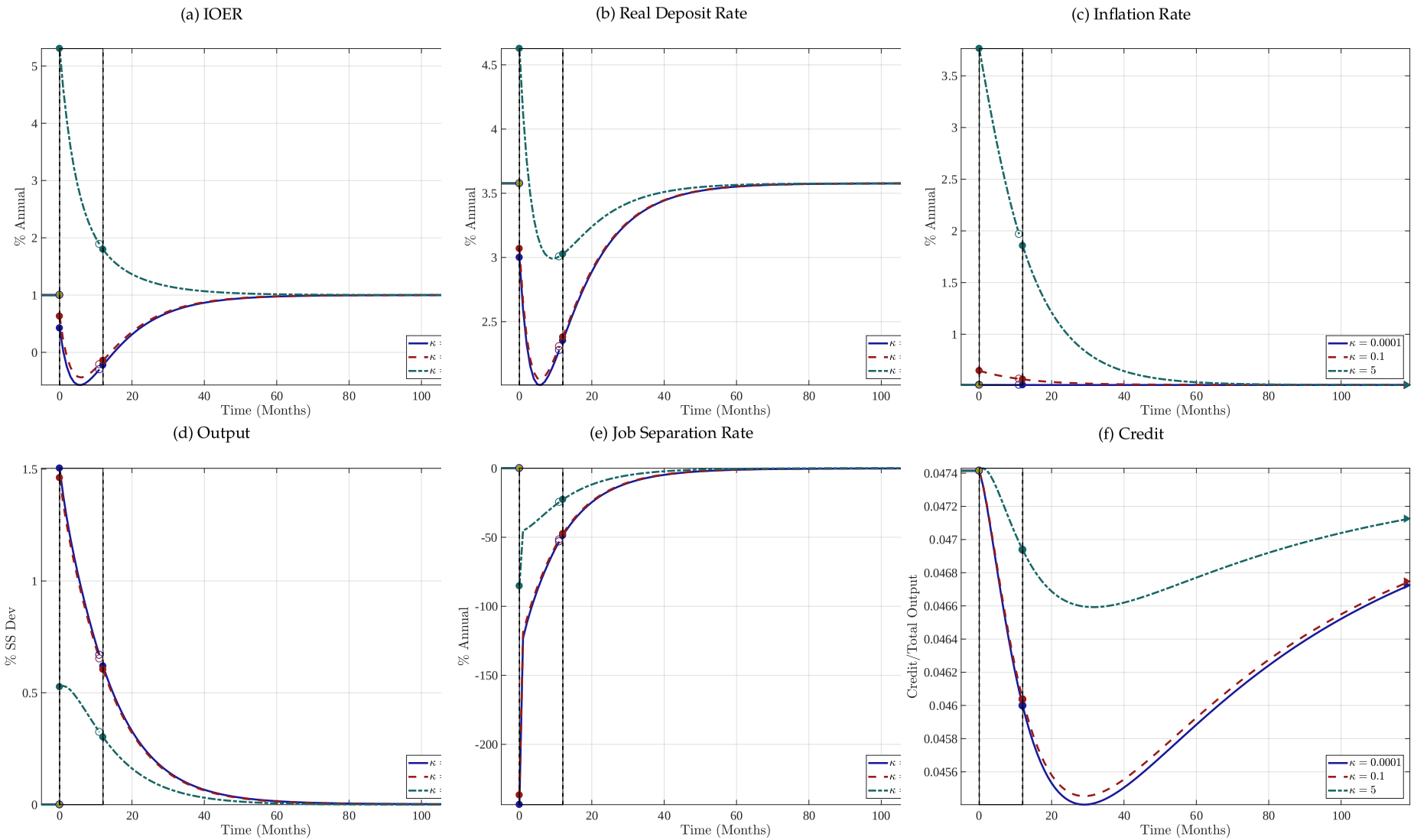


Figure 21: Transition Dynamics after a IOR Reduction (Interest Rate Target).

Note: The figure reports the responses to rates, inflation, output, job separation rate and credit after an unanticipated IOR reduction. In panels (a), (b), (c) and (e), all the rates are expressed in annual percentages. In panels (d), the aggregate output is expressed in percentage deviations from the steady state. In panel (f), the aggregate credit is expressed in credit-to-total output ratio. The reduction in the IOR is unanticipated at time zero (at the first vertical dashed line) and lasts for one year (between the two vertical dashed lines). Starting time 0, the path of IOR follows the modified Taylor Rule, where the short-run path of IOR is independent of the inflation rate . The IOR is initially fixed at 1% annually before time 0.

Effects of IOR beyond the DZLB. In the following figures we consider the following scenario of the Taylor Rule: $\eta_t \equiv \bar{\eta}$, and the following discretionary rate path:

$$\zeta_t^{LR} = \begin{cases} 0, & \text{if } t \in \left[T^{pre}, T^{post}\right] \\ \infty, & \text{otherwise} \end{cases}, \ \zeta_t^{SR} \equiv \infty,$$

This implies the following path of i_t^m:

$$i_t^m = \begin{cases} \bar{i}_0^m + \bar{\eta} \cdot \pi_t, & \text{if } t \in \left[T^{pre}, T^{post}\right] \\ i_\infty^m + \bar{\eta} \cdot \pi_t, & \text{otherwise} \end{cases}$$

This scenario is a bang-bang control of $\bar{i}_t^m$, where DZLB only occurs during $\left[T^{pre}, T^{post}\right]$. We do not change the Taylor Rule, but only uses an aggressive path of discretionary rate. There can be weak indirect GE effect through the path of inflation.

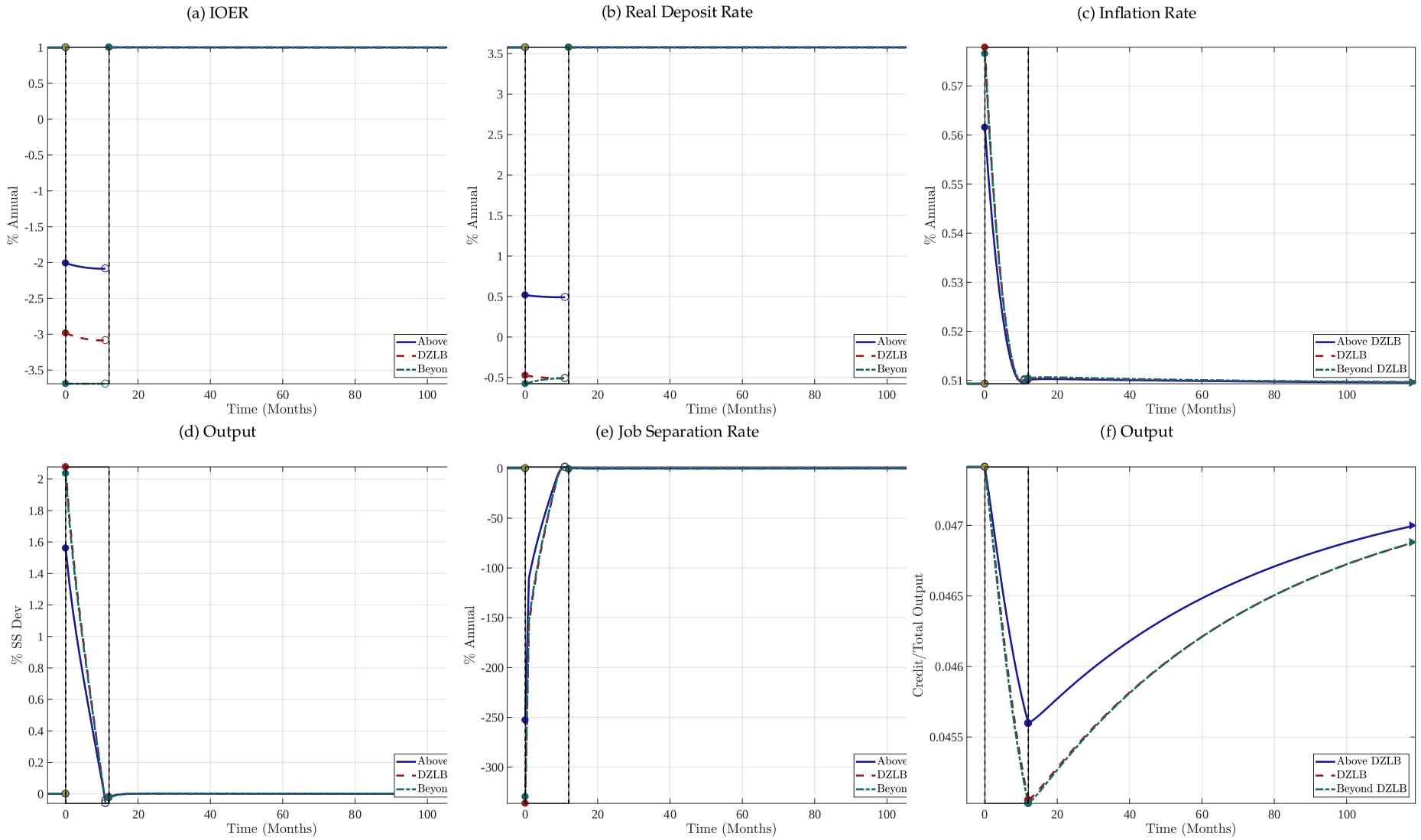


Figure 22: Transition Dynamics after a IOR Reduction (Negative IOR and DZLB).

Note: The figure reports the responses of rates, inflation, output, job separation rate and credit after an unanticipated IOR reduction. In panels (a), (b), (c) and (e), all the rates are expressed in annual percentages. In panel (d), the aggregate output is expressed in percentage deviations from the steady state. In panel (f), the aggregate credit is expressed in credit-to-total output ratio. The reduction in the IOR is unanticipated at time zero (at the first vertical dashed line) and lasts for one year (between the two vertical dashed lines). Starting from time 0, the path of IOR follows the modified Taylor Rule. The IOR is initially fixed at 1% annually before time 0. Scenario "DZLB" means the IOR is reduced to the level that reduces the nominal deposit rate to 0. Scenario "Above DZLB" means the IOR is reduced to the level higher than the DZLB level. Scenario "Beyond DZLB" means the IOR is reduced to the level lower than the DZLB level.

H Discount Factor Shocks

H.1 Simulation Results: Closing Spread During Discount Factor Shock

We simulate the model under $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. Note that the range of spread that can be implemented in the current calibration is $\Delta r \in [0.44\%, 1.26\%]$. The simulation of $\Delta r = 0$ has some convergence issue, so I did not present it here. In all the simulations below, the discount factor shock takes the form: $e^{-\rho t}\delta(t)$, where $\delta(t)$ is a U-shape curve over time, and assume $\delta(t) = 1$ if there is no discount factor shock. Figure 6 plots the path of $\delta(t)$. For the long-run monetary policy we set $i^m_{ss} = 1\%$ for all scenarios. For the monetary policy during shock, we set $\Delta r_t = 0$ and do not change i^m_t. The following table reports the welfare loss (in terms of certainty equivalence) at time 0 and the following figure plots the transition paths of all scenarios.

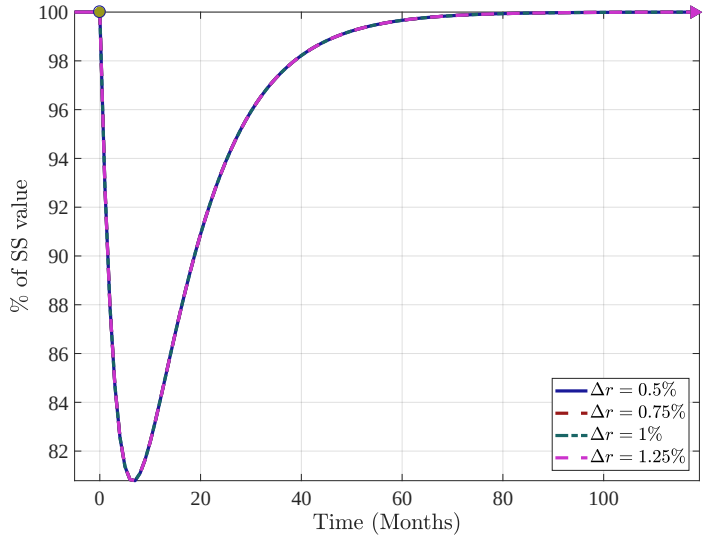
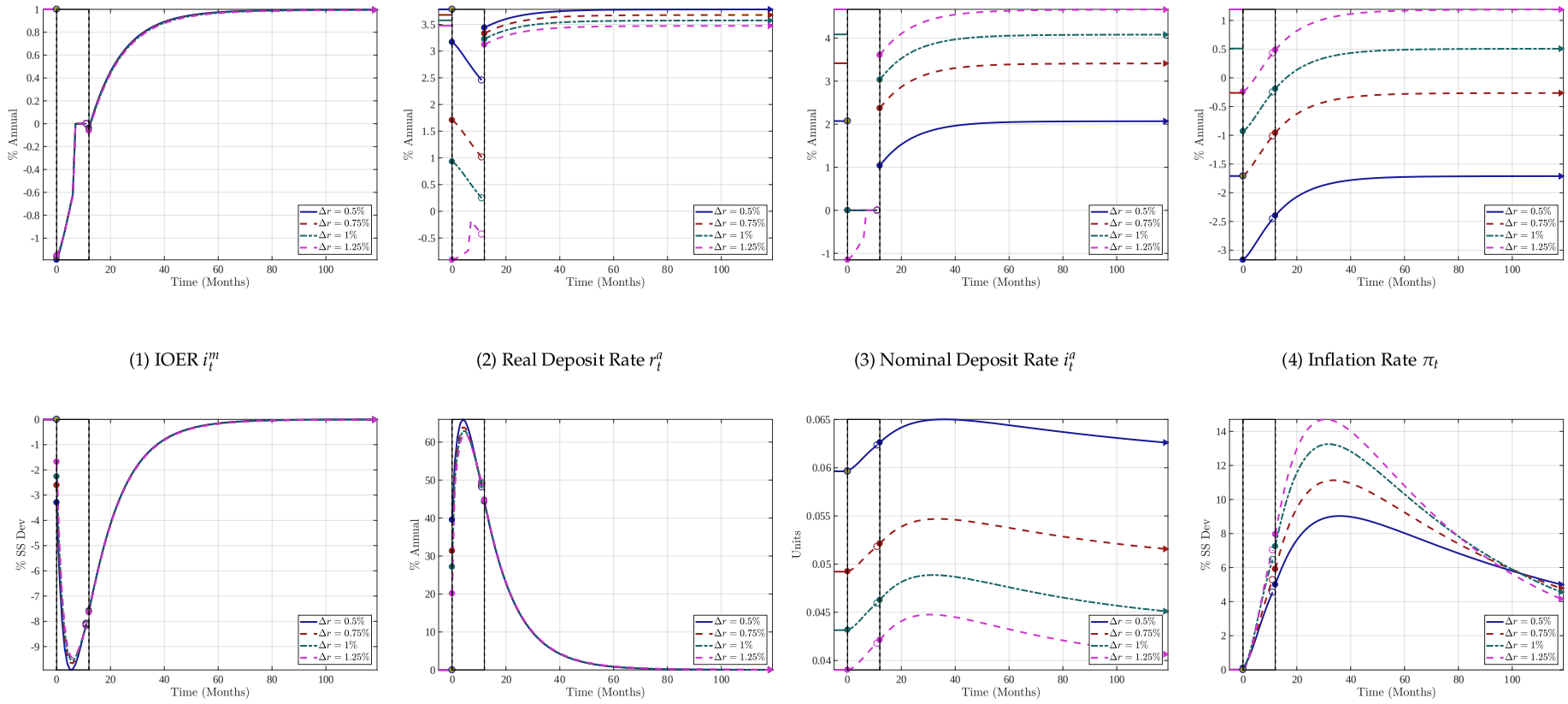


Figure 23: The Path of Discount Factor Shock $\delta(t)$

Note: The figure reports the path of discount factor shock $\delta(t)$. The shock enters households' per-period utility in the form of $\delta(t)U(c_t)$. In steady state we normalize $\delta(t) = 1$. During the shock, a $\delta(t) < 1$ represents a temporary increase in the discount rate. The values of $\delta(t)$ in the figure are expressed in percentages of the steady state value.

Table 6: Welfare Loss of Closing Spread During Discount Factor Shock

Scenario of Δr	0.5%	0.75%	1%	1.25%
Transition Path Welfare Loss (% deviation of CE from Y_{ss})	0.5350	0.5339	0.5368	0.5385
Steady State Welfare Loss (% deviation of CE from Y_{ss})	0.0238	0.0284	0.0326	0.0365



(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 24: Transition Paths of Closing Spread in Discount Factor Shock

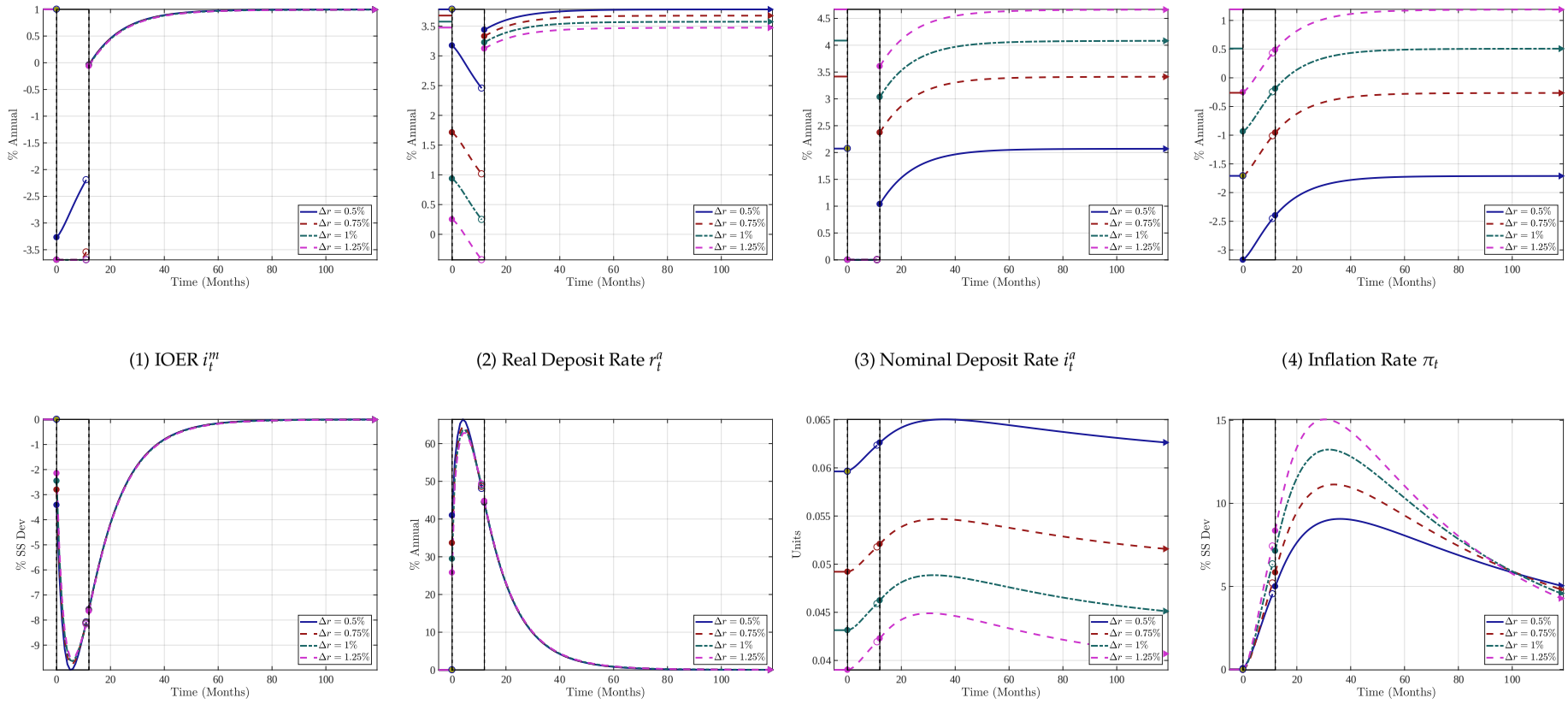
Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated discount factor shock and credit spread reduction. The credit spread is reduced to zero during credit crunch, and increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\bar{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set $i_{ss}^m = 1\%$ for all scenarios. For the monetary policy during credit crunch, we set $\Delta r_t = 0$ and do not change i_t^m. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

H.2 Simulation Results: Reducing IOER to DZLB During Discount Factor Shock

We simulate the model under $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. Note that the range of spread that can be implemented in the current calibration is $\Delta r \in [0.44\%, 1.26\%]$. The simulation of $\Delta r = 0$ has some convergence issue, so I did not present it here. In all the simulations below, the discount factor shock takes the form: $e^{-\rho t}\delta(t)$, where $\delta(t)$ is a U-shape curve over time, and assume $\delta(t) = 1$ if there is no discount factor shock. Figure 6 plots the path of $\delta(t)$. For the long-run monetary policy we set $i^m_{ss} = 1\%$ for all scenarios. For the monetary policy during shock, we set $\tilde{i}^m_0 = -\frac{1}{2}\left[\chi^+_{ss} + (1-\delta)\chi^-_{ss}\right]$ and do not change Δr_t, so the nominal deposit rate $i^a_t \equiv 0$ during shock. The following table reports the welfare loss (in terms of certainty equivalence) at time 0 and the following figure plots the transition paths of all scenarios.

Table 7: Welfare Loss of Reducing IOER to DZLB During Discount Factor Shock

Scenario of Δr	0.5%	0.75%	1%	1.25%
Transition Path Welfare Loss (% deviation of CE from Y_{ss})	0.5366	0.5365	0.5394	0.5426
Steady State Welfare Loss (% deviation of CE from Y_{ss})	0.0238	0.0284	0.0326	0.0365



(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 25: Transition Paths of Reducing IOER to DZLB during Discount Factor Shock

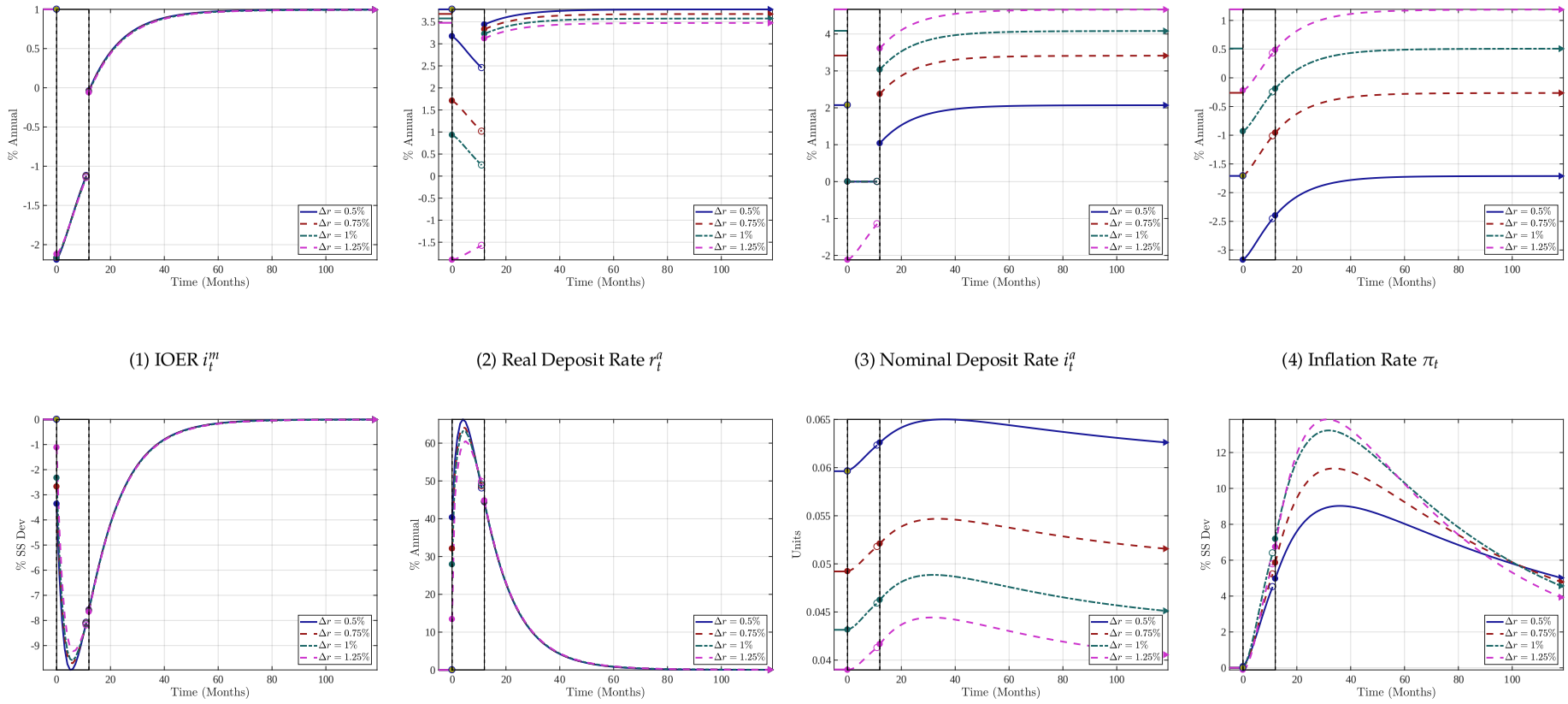
Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated discount factor shock and IOER reduction. During the credit crun we set $\bar{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\,\chi_{ss}^-\right]$ and do not change Δr_t. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\bar{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

H.3 Simulation Results: Reducing IOER to DZLB and Closing Spread During Discount Factor Shock

We simulate the model under $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. Note that the range of spread that can be implemented in the current calibration is $\Delta r \in [0.44\%, 1.26\%]$. The simulation of $\Delta r = 0$ has some convergence issue, so I did not present it here. In all the simulations below, the discount factor shock takes the form: $e^{-\rho t}\delta(t)$, where $\delta(t)$ is a U-shape curve over time, and assume $\delta(t) = 1$ if there is no discount factor shock. Figure 6 plots the path of $\delta(t)$. For the long-run monetary policy we set $i_{ss}^m = 1\%$ for all scenarios. For the monetary policy during shock, we set $\bar{i}_0^m = 0$ and $\Delta r_t = 0$, so the nominal deposit rate $i_t^a \equiv 0$ during this period. Out of the shock, $\Delta r_t \equiv \Delta r_{ss}$. The following table reports the welfare loss (in terms of certainty equivalence) at time 0 and the following figure plots the transition paths of all scenarios.

Table 8: Welfare Loss of Reducing IOER to DZLB and Closing Spread During Discount Factor Shock

Scenario of Δr	0.5%	0.75%	1%	1.25%
Transition Path Welfare Loss (% deviation of CE from Y_{ss})	0.5361	0.5349	0.5378	0.5308
Steady State Welfare Loss (% deviation of CE from Y_{ss})	0.0238	0.0284	0.0326	0.0365



(1) IOER i_t^m (2) Real Deposit Rate r_t^a (3) Nominal Deposit Rate i_t^a (4) Inflation Rate π_t

(5) Output (% Deviation from Steady State) (6) Job Separation Rate ϕ_t (7) Credit B_t (Level) (8) Credit B_t (% Deviation from Steady State)

Figure 26: Transition Paths of Reducing IOER to DZLB and Closing Spread During Discount Factor Shock

Note: The figure reports the paths of IOER, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated discount factor shock with a simultaneous reduction in credit spread and IOER. During the credit crun we set $\Delta r_t = \tilde{i}_0^m = 0$. Out of the credit crunch we set $\Delta r_t \equiv \Delta r_{ss}$ for all scenariosand increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\tilde{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

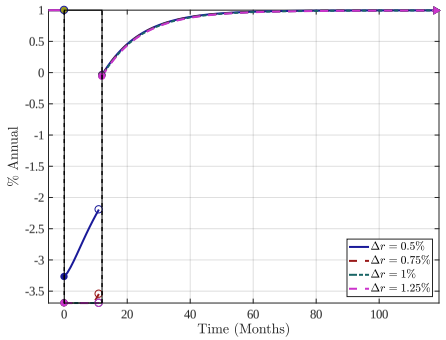
H.4 Simulation Results: Going Below DZLB Does No Good

We simulate the model under $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. Note that the range of spread that can be implemented in the current calibration is $\Delta r \in [0.44\%, 1.26\%]$. The simulation of $\Delta r = 0$ has some convergence issue, so I did not present it here. In all the simulations below, the discount factor shock takes the form: $e^{-\rho t}\delta(t)$, where $\delta(t)$ is a U-shape curve over time, and assume $\delta(t) = 1$ if there is no discount factor shock. Figure 6 plots the path of $\delta(t)$. For the long-run monetary policy we set $i^m_{ss} = 1\%$ for all scenarios. For the monetary policy during shock, we set $\tilde{i}^m_0 = -\frac{1}{2}\left[\chi^+_{ss} + (1-\delta)\chi^-_{ss}\right]$ and do not change Δr_t, so the nominal deposit rate $i^a_t \equiv 0$ during shock. This exercise is to answer whether going below DZLB does any good compared to going to exactly DZLB (the simulation we considered in section 3.2). So we compare the figures in section 3.2 and in this section, and plot them together as below. The following table reports the welfare loss (in terms of certainty equivalence) at time 0.

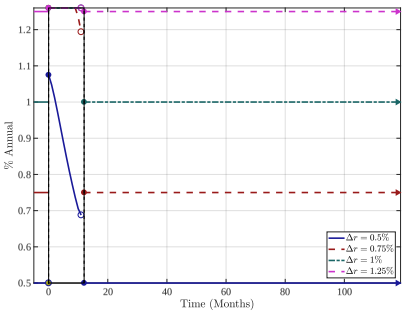
Table 9: Welfare Loss of Reducing IOER Below DZLB During Discount Factor Shock

Scenario of Δr	0.5%	0.75%	1%	1.25%
Transition Path Welfare Loss (% deviation of CE from Y_{ss})	0.5375	0.5366	0.5394	0.5426
Steady State Welfare Loss (% deviation of CE from Y_{ss})	0.0238	0.0284	0.0326	0.0365

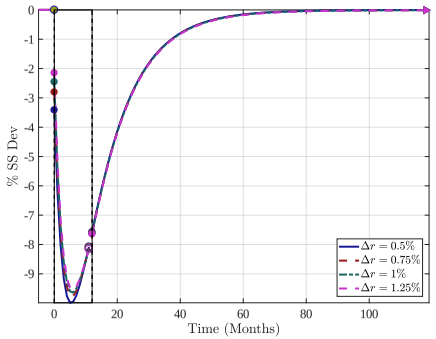
The left column of figures below plot the scenario of reducing IOER to DZLB (section 4.2), and the right column of figures plot the scenario of reducing IOER below DZLB (this section). One can see that the output decreases more during credit crunch in the right column compared to the left.



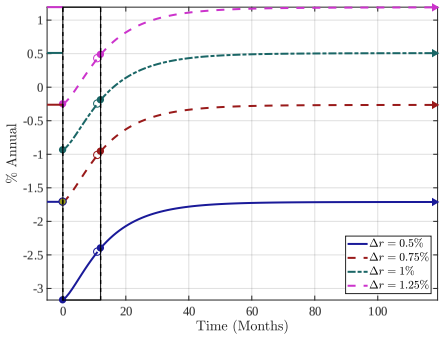
(1.a) IOER i_t^m (Reducing IOER to DZLB)



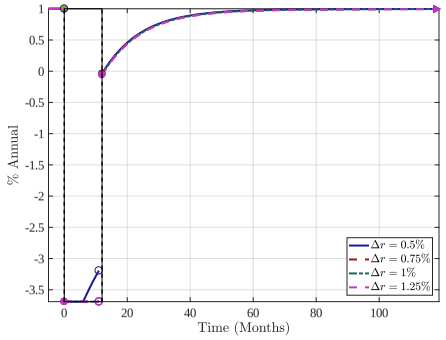
(2.a) Spread Δr_t (Reducing IOER to DZLB)



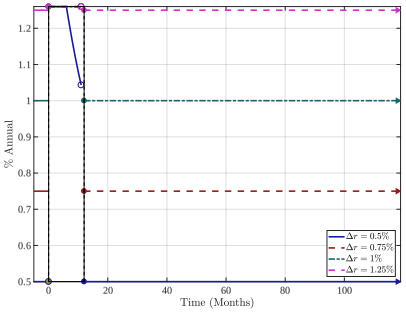
(3.a) Output (% Deviation from Steady State, Reducing IOER to DZLB)



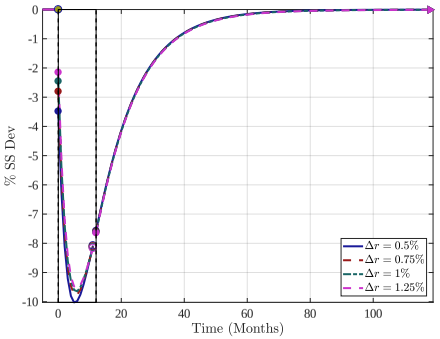
(4.a) Inflation Rate π_t (Reducing IOER to DZLB)



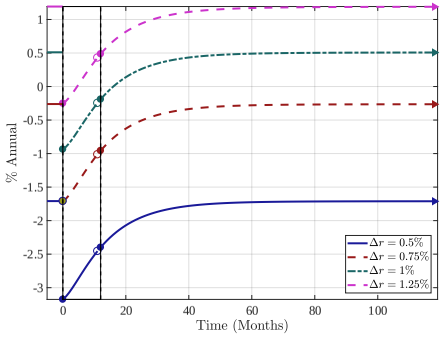
(1.b) IOER i_t^m (Reducing IOER below DZLB)



(2.b) Spread Δr_t (Reducing IOER below DZLB)



(3.b) Output (% Deviation from Steady State, Reducing IOER below DZLB)



(4.b) Inflation Rate π_t (Reducing IOER below DZLB)

Figure 27: Transition Paths of Reducing IOER to DZLB vs Reducing IOER below DZLB

Note: The figure reports the paths of IOER, credit spread, aggregate output and inflation rate after an unanticipated discount factor shock and IOER reduction. In the panels on the left column, we set $\bar{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\chi_{ss}^-\right]$ and do not change Δr_t during credit crunch. In the panels on the right column, we set $\bar{i}_0^m = -\frac{1}{2}\left[\chi_{ss}^+ + (1-\delta)\chi_{ss}^-\right]$-$\Delta i_0$ and changes Δr_t such that i_t^a does not fall below zero during credit crunch. The value of $\Delta i_0 = \min\left\{0.5\%, \frac{1}{2}\left[\chi^+(+\infty) - \chi_{ss}^+ + (1-\delta)\left(\chi^-(+\infty) - \chi_{ss}^-\right)\right]\right\}$. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\bar{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (3.a) and (3.b), the aggregate output is expressed in percentage deviations from the steady state.

H.5 Welfare Comparison

The following table compares the transition path welfare loss across policies

Table 10: Welfare Loss of Policies During Discount Factor Shock (% deviation of CE from Y_{ss})

Policy Scenario	Steady-State Spread Δr_{ss}			
	0.5%	0.75%	1%	1.25%
Close Spread	0.5350	0.5339	0.5368	0.5385
Reduce IOER to DZLB	0.5366	0.5365	0.5394	0.5426
Reduce IOER to DZLB and Close Spread	0.5361	0.5349	0.5378	0.5308
Reduce IOER Below DZLB	0.5375	0.5366	0.5394	0.5426